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Using Optimization to Construct Naturally Parametrized Curve with Cubic Curvature

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1 INTRODUCTION AND MAJOR CHALLENGES

Naturally parameterized smooth curves are used widely in practice where it is desirable to describe flow, smooth trajectory, path or shapes derived from thereof. The derived shapes are, for example, aerodynamic and hydrodynamics cowls, nozzles, wings, propellers and turbine blades.

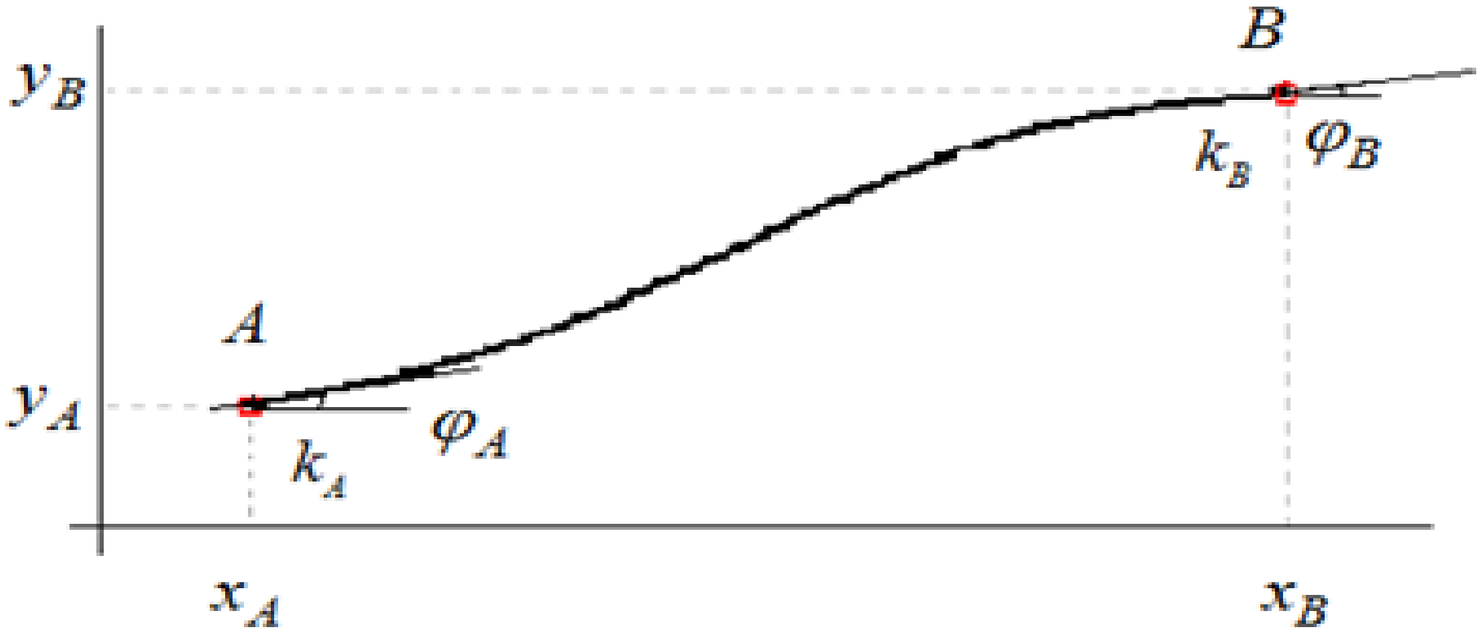
It is well known, that any unplanned non-smoothness in aerodynamic surface can cause aerodynamic resistance. This is even more important in transsonic mode. Consequently, specialized methods for building flow-like curves should have inbuilt capacity to limit surface curvature and provide smooth interfaces.

2 AIM AND RESEARCH TASKS

General aim of research this work is part of, is construction of naturally parameterized curves which would allow describing hydro- and aerodynamic shapes with minimum parameters.

Suggested approach for this work is solution for the next problem: it is required to connect the points two-dimensional curve in natural parameterization using cubic law of curvature distribution (the curve is determined by the length S, where the curvature $k(s) = as^3 + bs^2 + cs + d$ has a cubic dependence on the length of the curve) so that at the points A and B the given values of tangent angles φ_A, φ_B and curvature values k_A, k_B were achieved.

The geometric model of the problem



3 RESEARCH METHODOLOGY

The overall research methodology for this work is limited in scope to finding the method of building curve and testing it [1].

[1]. Stetsyuk, P.I., Tkachenko, O.V., Zhydkov, V.O.: Using Shor's r-algorithm for building naturally parametrized curve having cubic curvature. *Proceedings of the 7-th International Conference on Control and Optimization with Industrial Application*, Baku, Azerbaijan, 26-28 August, Vol. I. 389-391 (2020).

The System of Nonlinear Integral Equations

$$x_B = x_A + \int_0^S cos\left(\varphi_A + \frac{as^4}{4} + \frac{bs^3}{3} + \frac{cs^2}{2} + d \times s\right) ds\,, \tag{1}$$

$$y_B = y_A + \int_0^S sin\left(\varphi_A + \frac{as^4}{4} + \frac{bs^3}{3} + \frac{cs^2}{2} + d \times s\right) ds\,, \tag{2}$$

$$\varphi_B = \varphi_A + \frac{aS^4}{4} + \frac{bS^3}{3} + \frac{cS^2}{2} + d \times S, \tag{3}$$

$$k_A = d, \quad k_B = aS^3 + bS^2 + cS + d. \tag{4}$$

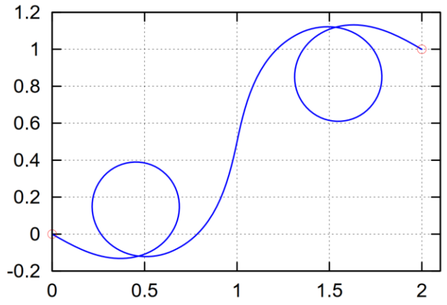
Optimization Problem: unknown $a^\star, b^\star, c^\star, S^\star$

$$f^\star = f(a^\star, b^\star, c^\star, S^\star) = \min_{a,b,c,S} \left\{ f(a,b,c,S) = \sum_{i=1}^{4} |f_i(a,b,c,S)| \right\} \qquad (5)$$

$$S_{\min} \le S \le S_{\max}, \qquad (6)$$

$$-\frac{\pi}{2} \le \varphi_A + a\frac{i^4 S^4}{4N^4} + b\frac{i^3 S^3}{3N^3} + c\frac{i^2 S^2}{2N^2} + k_A\frac{iS}{N} \le \frac{\pi}{2}, \quad i = 1, \dots, N, \qquad (7)$$

Constraint (7) ensures the existence of the unique global minimum for problem (5)–(7), absence of loops like this:



The residuals for equations (1)–(4)

$$f_1(a,b,c,S) = x_B - x_A - \int_0^S \cos\left(\varphi_A + \frac{as^4}{4} + \frac{bs^3}{3} + \frac{cs^2}{2} + k_A s\right) ds\,, \tag{8}$$

$$f_2(a,b,c,S) = y_B - y_A - \int_0^S \sin\left(\varphi_A + \frac{as^4}{4} + \frac{bs^3}{3} + \frac{cs^2}{2} + k_A s\right) ds, \tag{9}$$

$$f_3(a,b,c,S) = \varphi_B - \varphi_A - \frac{aS^4}{4} - \frac{bS^3}{3} - \frac{cS^2}{2} - k_A S, \tag{10}$$

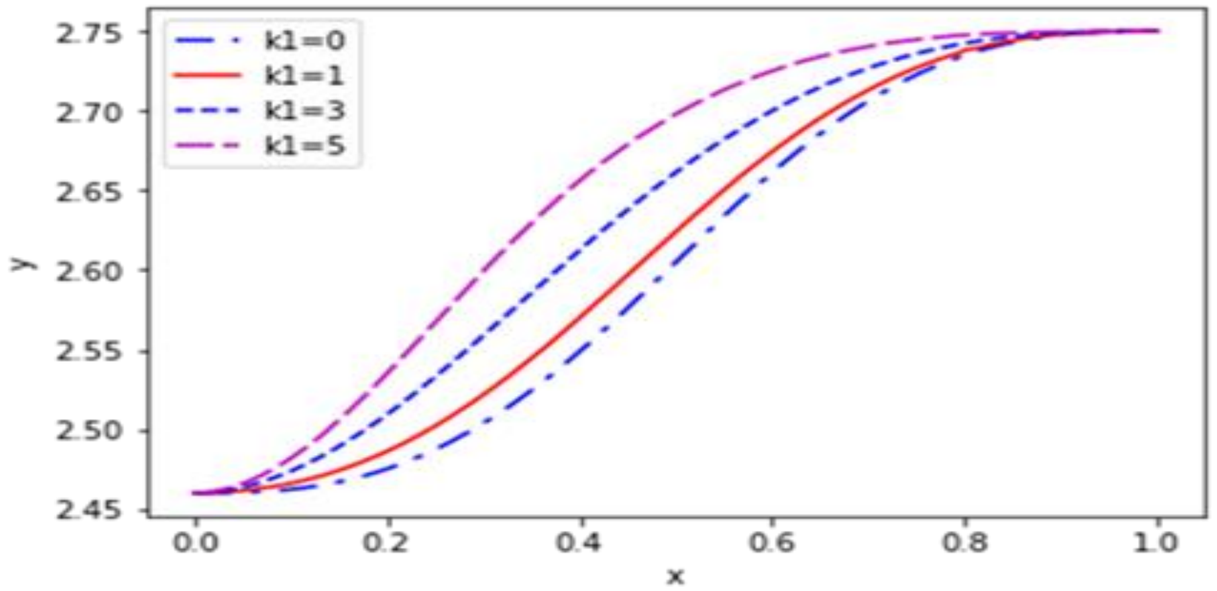
$$f_4\left(a,b,c,s_p\right) = k_B - aS^3 - bS^2 - cS - k_A \tag{11}$$

Problem (5)–(7) is a problem of minimization of non-smooth function, which is defined not for all values S, but only for those that are positive and allow to calculate definite integrals for functions $f_1(a, b, c, S)$ and $f_2(a, b, c, S)$.

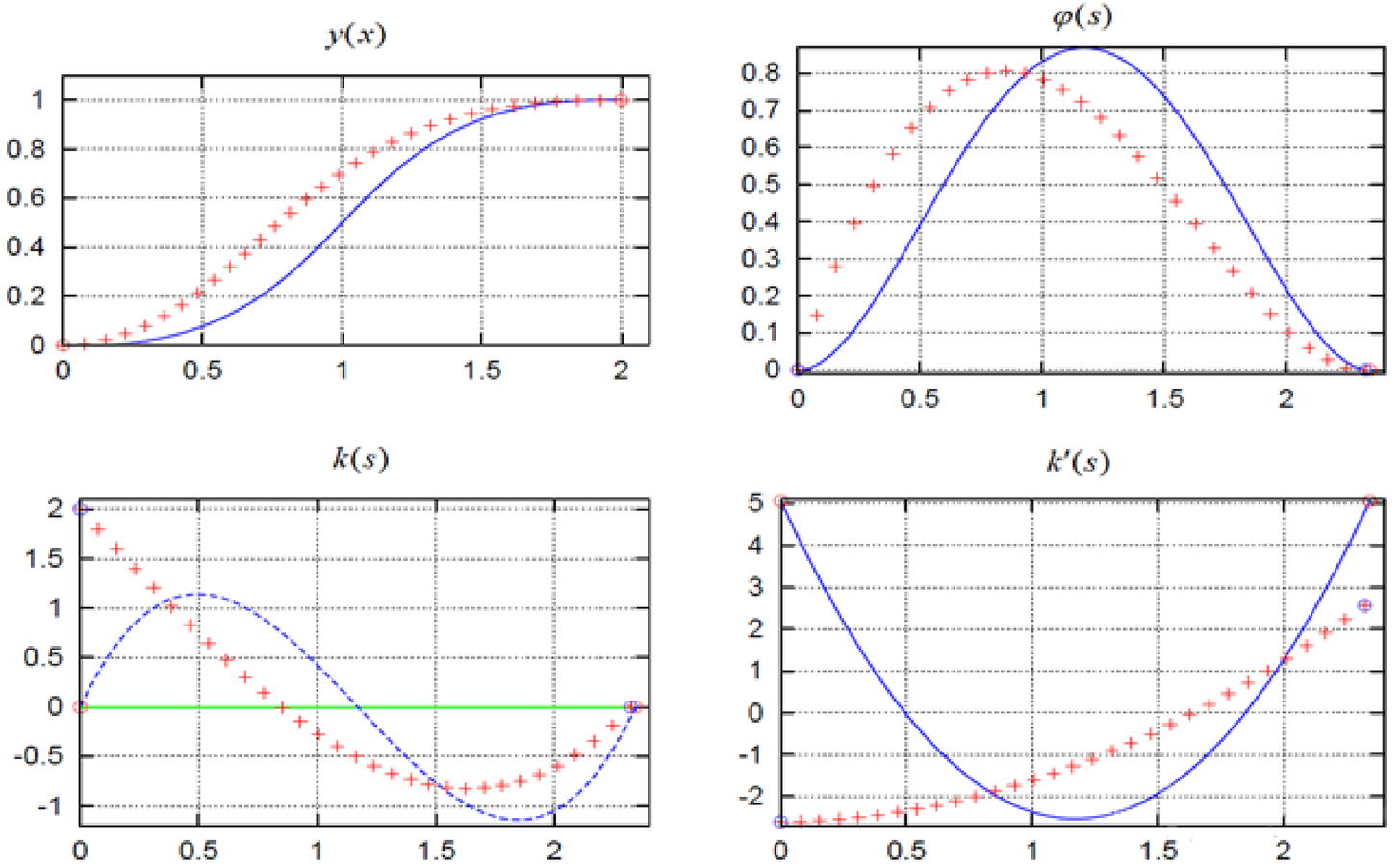
Algorithm for solving (5)-(7) was implemented here using modification of r-algorithm ralgb5a [2]. It is very efficient at finding minimum of the objective function.

[2]. Stetsyuk, P.I.: Theory and software implementations of Shor's r-algorithms. *Cybernetics and Systems Analysis* 53 (4), 692-703 (2017).

4 RESULTS



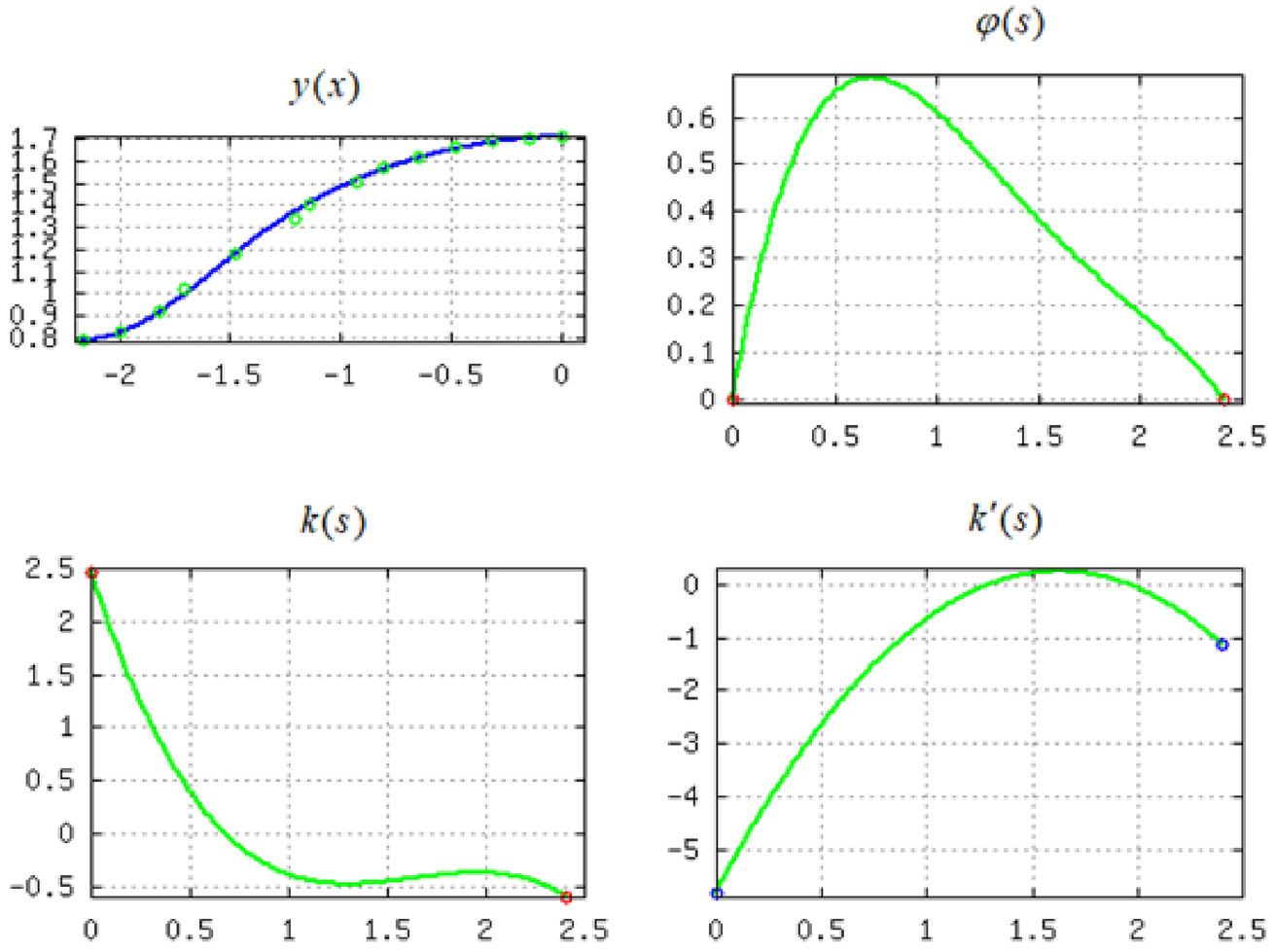
Fragments of the outer contour of the nozzle for different k_1



Two S-curves for the system (1)–(4): $k_1 = 0$ and $k_1 = 2$

Table 1. Point-defined contour, which includes 14 points

i	x_i	y_i	i	x_i	y_i
1	-216.680000	78.840891	8	-93.151429	150.726455
2	-199.388394	82.662538	9	-81.003369	156.694929
3	-182.333640	92.146215	10	-64.809795	162.158593
4	-170.482179	102.580692	11	-48.383973	166.193093
5	-147.849809	117.679087	12	-31.324910	168.943913
6	-120.476549	134.114910	13	-14.712846	170.439555
7	-93.151429	140.646664	14	0.000000	170.830030



Approximating curve with cubic curvature distribution law

5 CONCLUSIONS

The method can be used to profile varius hydro- and aero-dynamic surfaces, including transition channels of variable cross-section with the required geometric properties (along with the Bezier-Bernstein method).

It can be used to design S-shaped fragments of the outer contour of the nozzle and the contour of the central body. The choice of curve parameters allows to control the shape of the curve in such a way that at the base (reference) points the characteristics of the curve correspond to the specified characteristics of the projected profile.
When properly utilized, this can be a powerful tool for flow-based design.

ACKNOWLEDGMENT

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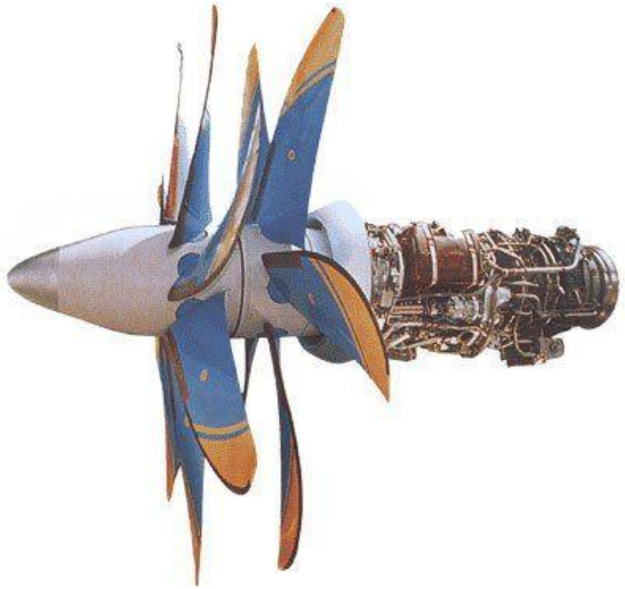
Ивченко прогресс Ivchenko PROGRESS



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