Using Shor's r-algorithm for building naturally parametrized curve having cubic curvature

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Outline

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Problem setting

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It is required to connect points $A(x_A, y_A)$ and $B(x_B, y_B)$ by two-dimensional naturally parametrized curve [1] (a curve is defined by its length S, its curvature has cubic dependence on length $k(s) = as^3 + bs^2 + cs + d$) in such a way that certain tangent angles φ_A, φ_B and curvature values k_A, k_B at points A and B are provided.

Problem background: aerodynamics

Smooth surfaces

It is well known, that any non-smoothness in aerodynamic surface can cause aerodynamic resistance. The situation worsens in transsonic mode. Consequtively, method for building curves should have inbuilt capacity to limit surface curvature and provide smooth interfaces.

Sonic boom

Onwards from Sears-Haack research, it is known that for sonic boom minimization, cross-section distribution should satisfy certain limitations, which are best imposed as curvature distribution.

Equation system: $d^\star = k_A$

$$x_B = x_A + \int_0^S \cos\left(\varphi_A + \frac{as^4}{4} + \frac{bs^3}{3} + \frac{cs^2}{2} + d\times s\right) ds, \qquad (1)$$

$$y_B = y_A + \int_0^S \sin\left(\varphi_A + \frac{as^4}{4} + \frac{bs^3}{3} + \frac{cs^2}{2} + d\times s\right) ds, \qquad (2)$$

$$\varphi_B = \varphi_A + \frac{aS^4}{4} + \frac{bS^3}{3} + \frac{cS^2}{2} + dS, \qquad (3)$$

$$k(0) = d = k_A, \quad k(S) = aS^3 + bS^2 + cS + d = k_B, \qquad (4)$$

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Optimization Problem: unknown $a^\star$, $b^\star$, $c^\star$, $S^\star$

$$f^\star = f(a^\star, b^\star, c^\star, S^\star) = \min_{a,b,c,S} \sum_{i=1}^{4} |f_i(a, d, c, S)|, \tag{5}$$

$$S \geq \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}, \tag{6}$$

$$f_1(a, b, c, S) = x_B - x_A - \int_0^S \cos\left(\varphi_A + \frac{as^4}{4} + \frac{bs^3}{3} + \frac{cs^2}{2} + k_A s\right) ds,$$

$$f_2(a, b, c, S) = y_B - y_A - \int_0^S \sin\left(\varphi_A + \frac{as^4}{4} + \frac{bs^3}{3} + \frac{cs^2}{2} + k_A s\right) ds$$

$$f_3(a, b, c, S) = \varphi_B - \varphi_A - \frac{aS^4}{4} - \frac{bS^3}{3} - \frac{cS^2}{2} - k_A S,$$

$$f_4(a, b, c, S) = aS^3 + bS^2 + cS + k_A - k_B.$$

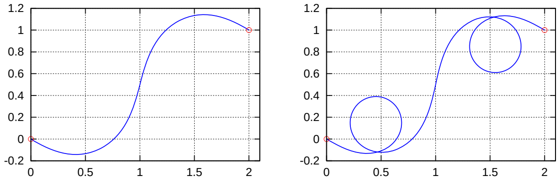
Algorithm 1: finding parameters problem

The problem (5)–(6) is multiextremal. Global minima coincide with solutions of the equation system (1)–(4)

Method: Shor's r-algorithm

Algorithm 1 for solving (5)–(6) was implemented using the multistart method and octave-function **ralgb5a** [3]. It finds the best local minimum of the objective function using a modification of r-algorithm and running it from predefined number of starting points.

Algorithm 1: multiple solutions



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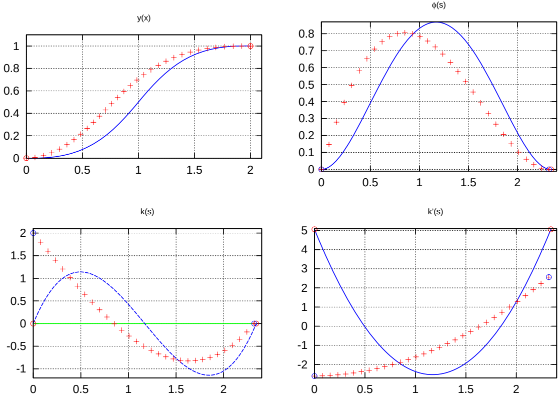
Algorithm 2: purpose

Algorithm 2

is intended to finding a single solution of the system (1)–(4) not containing cycles. It is based on utilizing the problem (5)–(6) complemented by an additional constraint

$$-\frac{\pi}{2} \leq \varphi(s) = \varphi_A + \frac{as^4}{4} + \frac{bs^3}{3} + \frac{cs^2}{2} + k_A s \leq \frac{\pi}{2} \quad \forall s \in [0, S],$$

Algorithm 2: results



Algorithm 2: application

Algorithm 2

is used for building fragments of technical profiles in the applications package "Surface", which is developed for making mathematical models of complicated curved surfaces. S-like curves are aimed to incorporate in the package "Soplo" for building geometric models of de Laval nozzle with a central body [4].

References:

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Questions?

THANK YOU FOR ATTENTION!