Optimization Problems for regular image reconstruction

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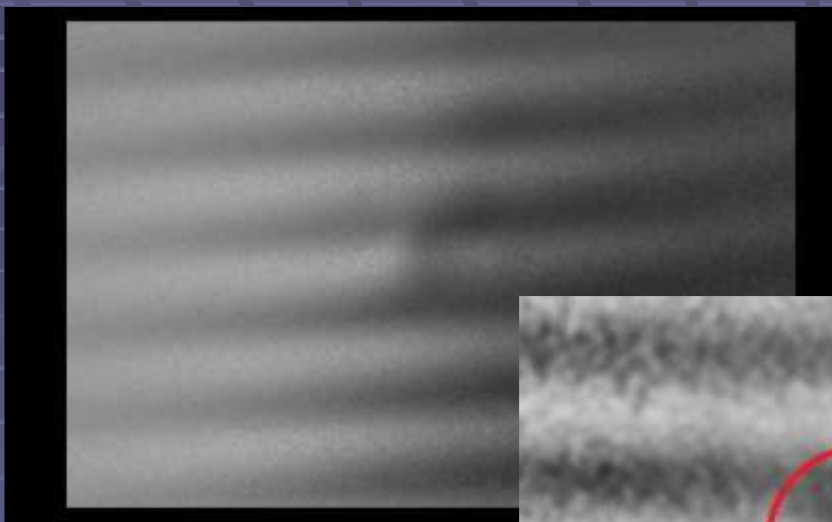
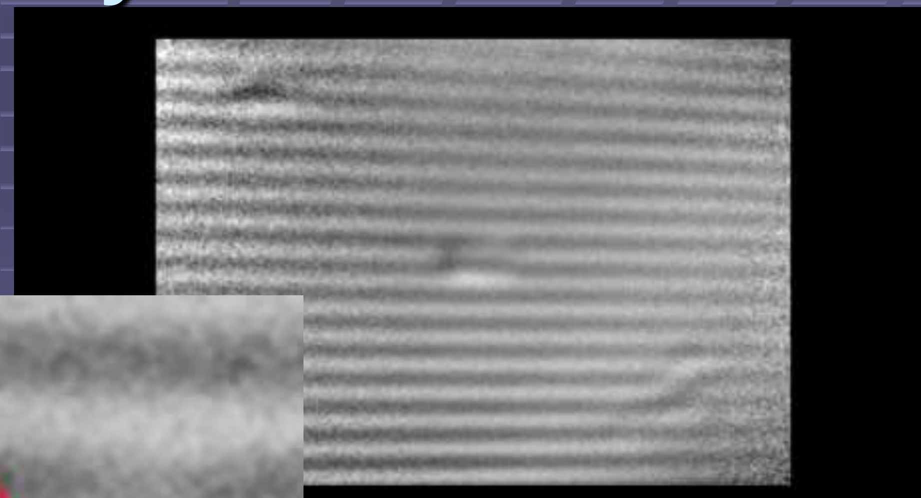
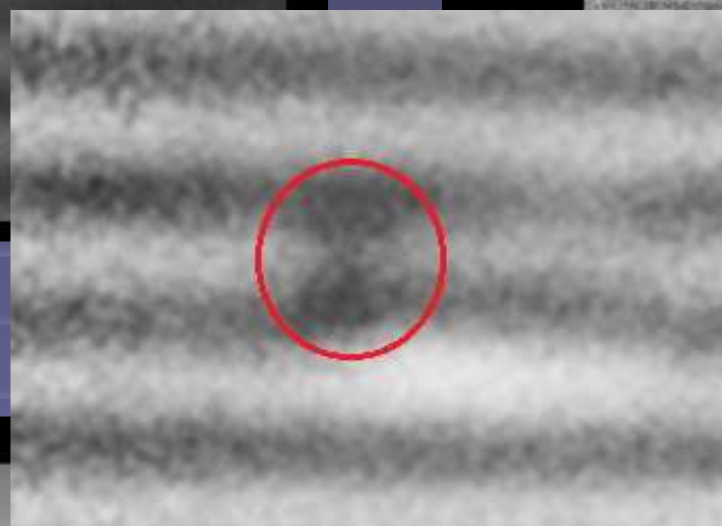
Outline

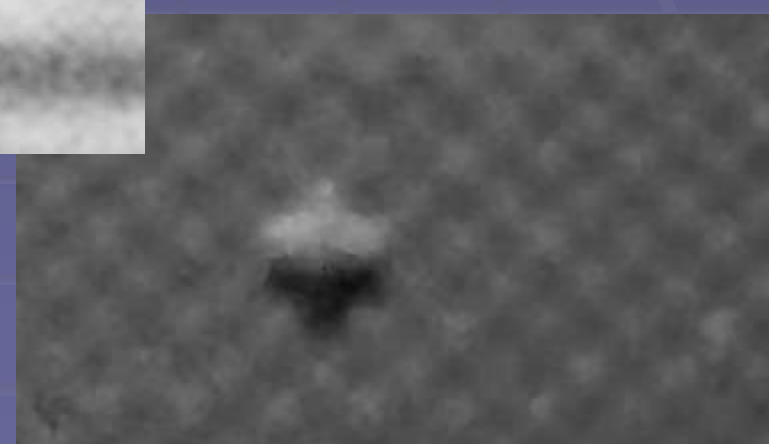
- 2D image of some regular backgroung with defects
- Defects take no more than 5-20% of area
- Problem: reconstruct periodic background and find defects
- Numerical experiments to estimate method robustness

Image examples: speckl-interferometry results







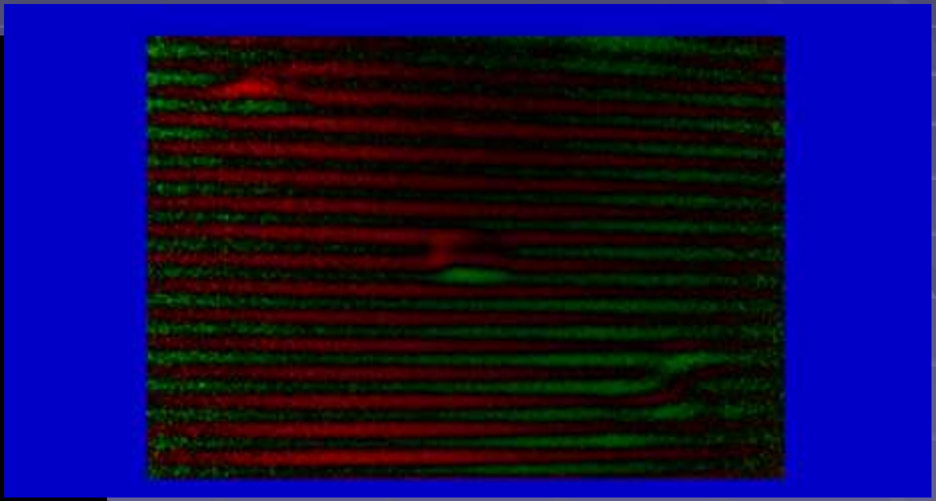


Suggested approach to problem solution

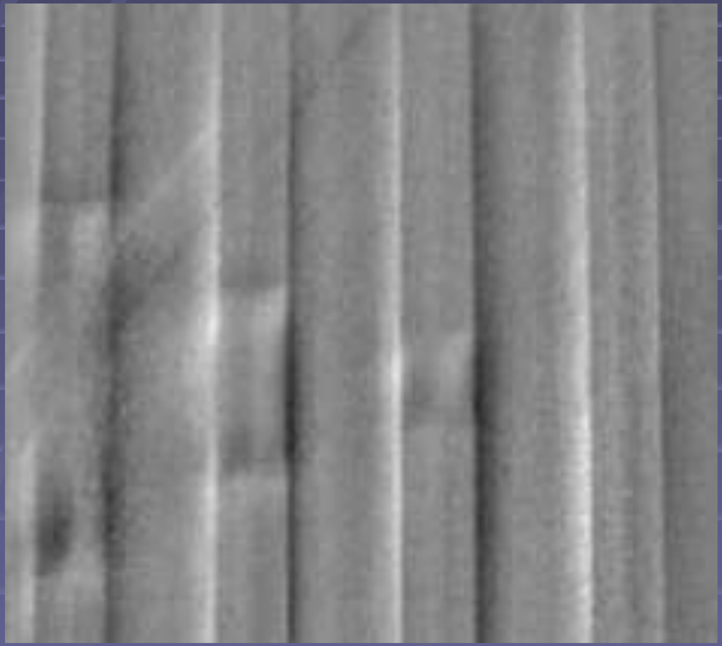
- Preliminary processing
- Constructing background model using optimization methods
- Substracting background

1. Preliminary processing

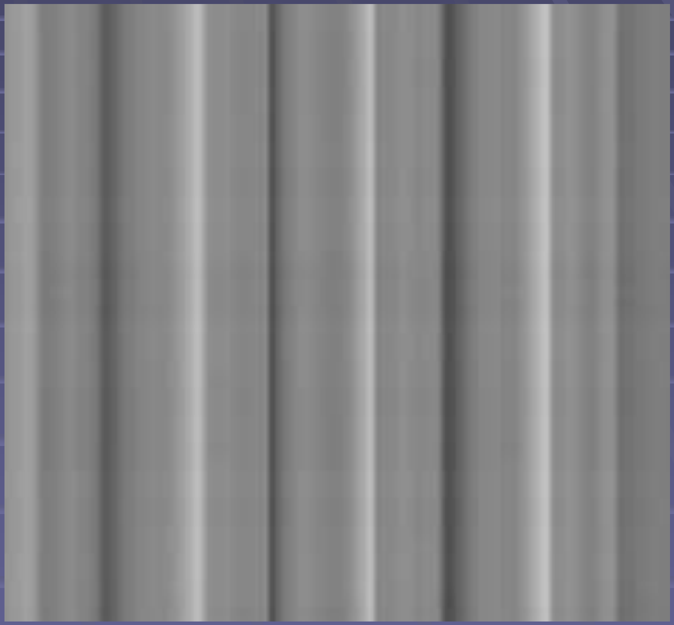
1.1Compensating for variations of background



Regular image examples



Regular with defects



Regular without defects

Regular 2D-image and its parameters

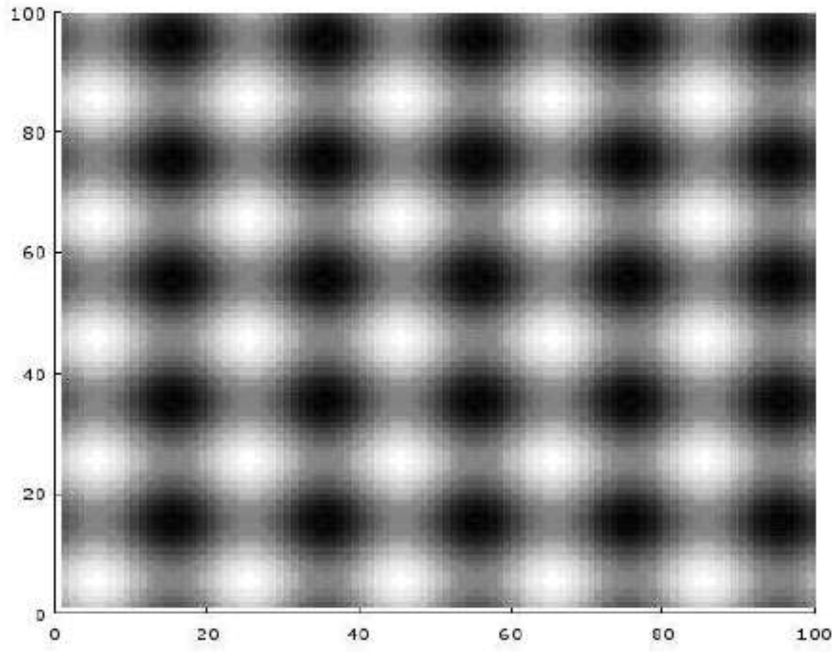
2D-image: triplet $\{A, u, v\}$ where

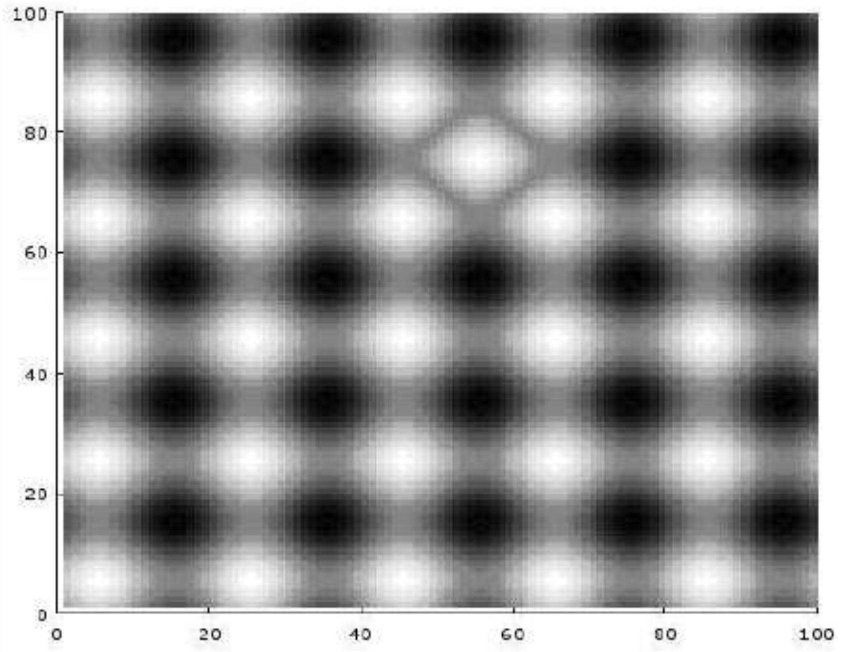
$$a_{ij} = u_i + v_j, u \in \Re^m v \in \Re^n$$

Regular 2D-image: $A = \{a_{ij}\}_{j=1,n}^{i=1,m} A \in \Re^{m \times n}$

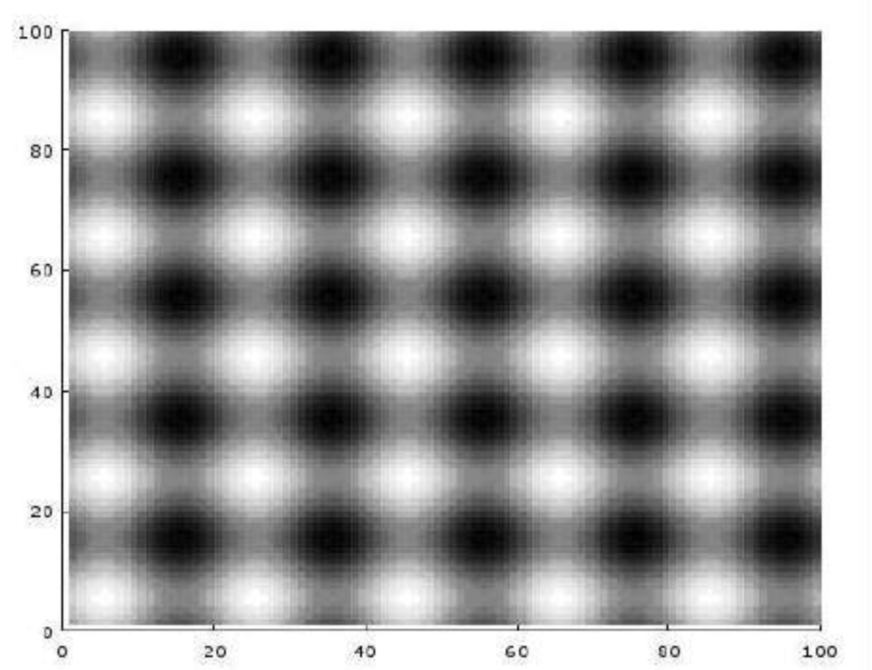
Defect in regular 2D-image: $\exists i \in 1...m, j \in 1...n, a_{ij} \neq u_i + v_j$

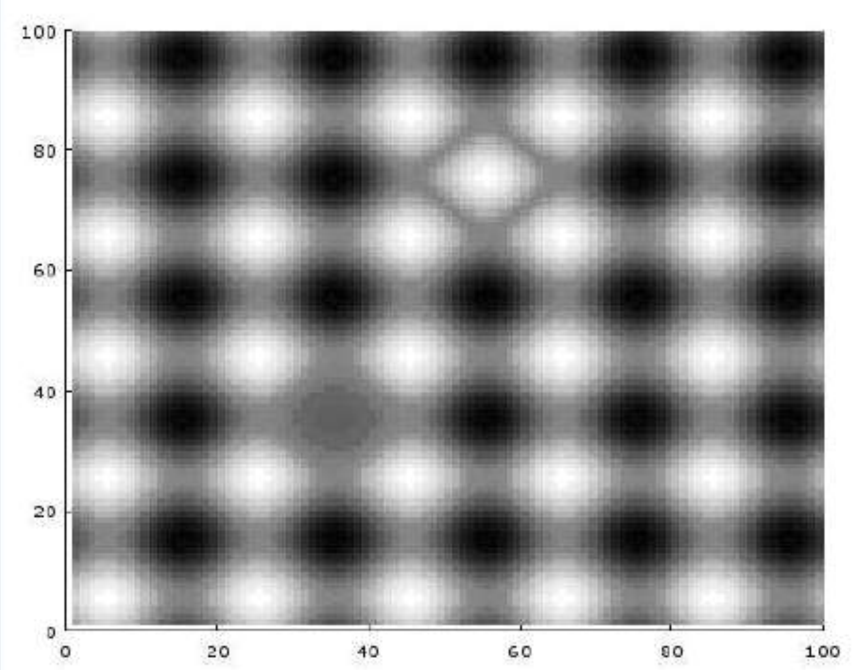
3D-structures, regular and with single defect





3D-structure, regular and with 2 defects





2 problems for regular 3D-structures

Problem 1 [2]. We have $m \times n$-matrix A. We need to find such vectors $x^* \in R^m$ and $y^* \in R^n$ for regular 3D-structure $\{A^*, x^*, y^*\}$ that coefficients of $m \times n$-matrix A^* had minimum deviation from coefficients of $m \times n$-matrix A in L_p-norm $(1 \leq p \leq 2)$.

Problem 2 [2]. We have $m \times n$-matrix A. For regular basis 3D-structure $\{A^{**}, x^{**}, y^{**}\}$ we need to find such vectors $x^{**} \in R^m$ and $y^{**} \in R^n$ that coefficients of matrix A^{**} had minimum deviation from coefficients of $m \times n$-matrix$A$.

Formulation 1 and 2

Formulation 1: find

$$(x^*, y^*) = \underset{\substack{x\in\Re^m \\ y\in\Re^n}}{\arg\min} \left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} \left| a_{ij} - x_i - y_j \right|^p \right\}$$

Formulation 2: find

$$(x^{**}, y^{**}) = \underset{\substack{x\in\Re^m \\ y\in\Re^n}}{\arg\min} \left\{ \sum_{i=1}^{m} \sum_{j=1}^{n} \left| a_{ij} - x_i - y_j \right|^p + \left| \sum_{i=1}^{m} x_i - \sum_{j=1}^{n} y_j \right|^p \right\}$$

Test examples and parameters of r-algorithm

Test example: pseudorandom noise

R-algorithm start point: $x_0 = (0, 0, \ldots, 0)^T$

Parameters: $\alpha = 2,\ h_0 = 1,\ q_2 = 1.1,\ n_h = 3$

Stop parameters $\varepsilon_x = 10^{-7},\ \varepsilon_g = 10^{-12}$

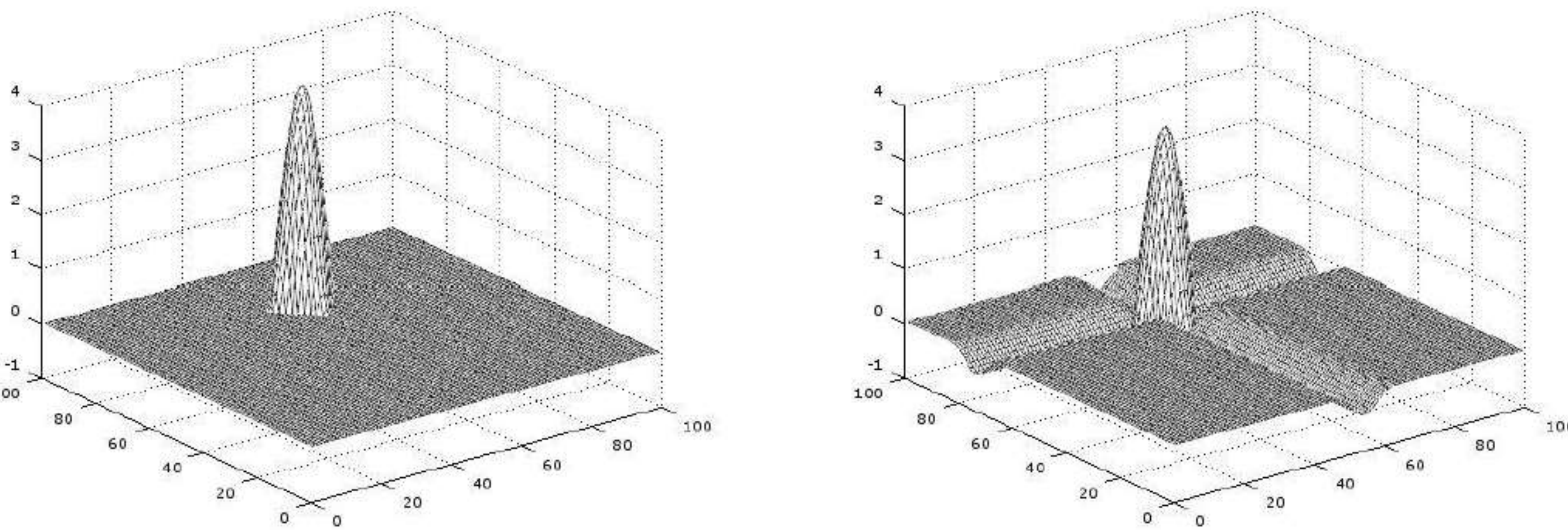
LSQ time for finding parameters of 3D regular 400x600 structure

№	Problem 1			Problem 2		
	itn	nfg	time	itn	nfg	time
1	304	518	7.60	302	512	7.52
2	278	468	6.88	285	475	7.05
3	266	441	6.54	277	461	6.83
4	268	445	6.68	295	494	7.78
5	269	449	6.74	284	477	7.16

LM times for finding parameters of 3D regular 400x600 structure

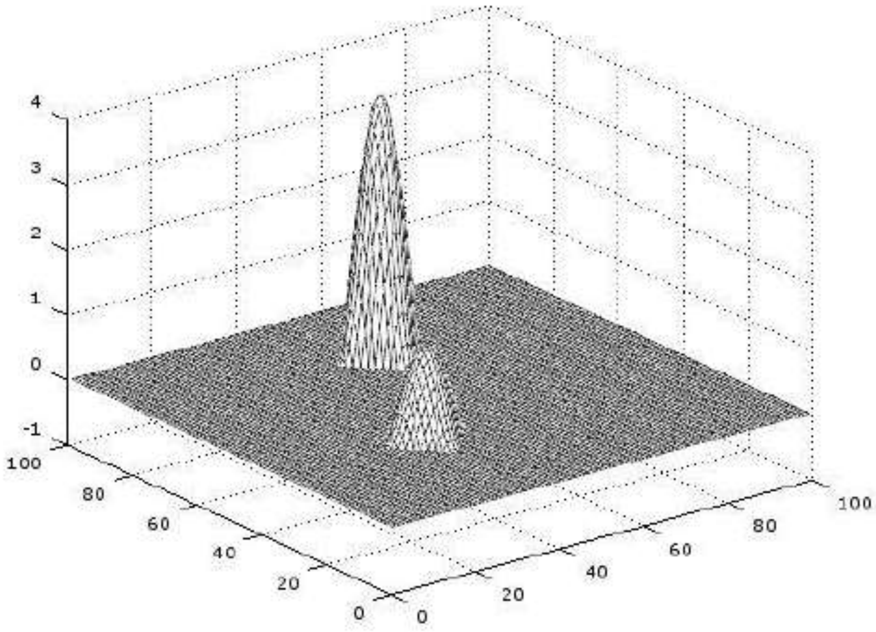
№	Problem 1			Problem 2		
	itn	nfg	t(sec)	itn	nfg	t(sec)
1	358	434	10.41	358	434	10.44
2	359	437	10.47	359	437	10.48
3	360	436	10.45	360	436	10.42
4	359	437	12.68	359	437	10.62
5	360	437	10.76	360	437	10.65

Robustness of LSQ and LM for finding single defect

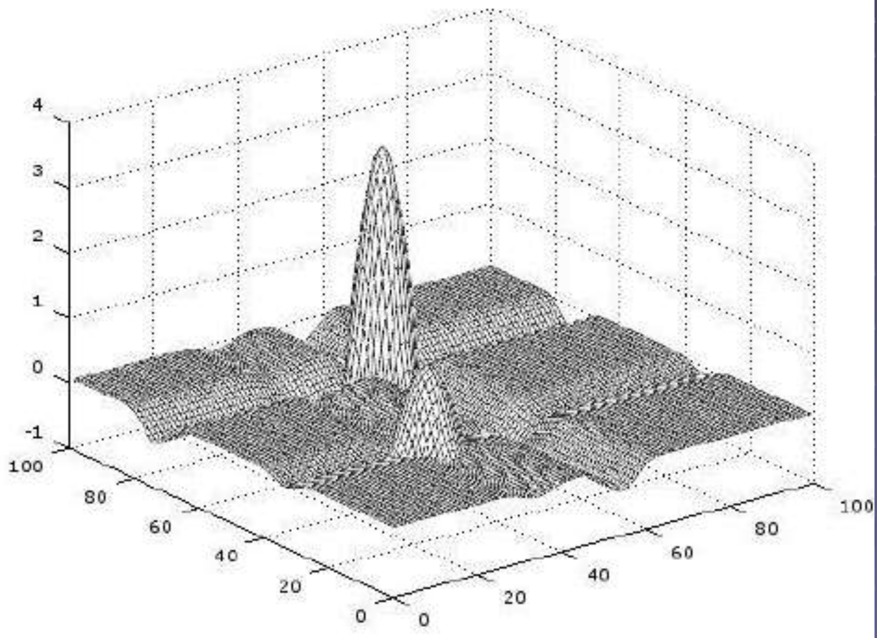


LM LSQ

Robustness of LSQ and LM for finding 2 defects



LM



LSQ

Conclusions

The method can be used for non-destructive quality control without human operator

R-algorithm is adequate solver for image regularization

The time for processing 400x600 images is adequate even for non-optimal program environment

Also, method allows optimization methods development for revealing defects in periodic 3D-structures of arbitrary shape.

Future plans:

- Processing images of arbitrary shape
- Processing in Fourier space
- Advanced error functions

Thanks

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