Two-Stage Transportation Problem with Unknown Consumer Demands: Consistency Aspects

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1. Two-stage Transportation Problem (TTP) with unknown consumer demands
2. Consistency conditions of TTP with unknown consumer demands
3. TTP with unknown consumer demands and fixed number of consumers

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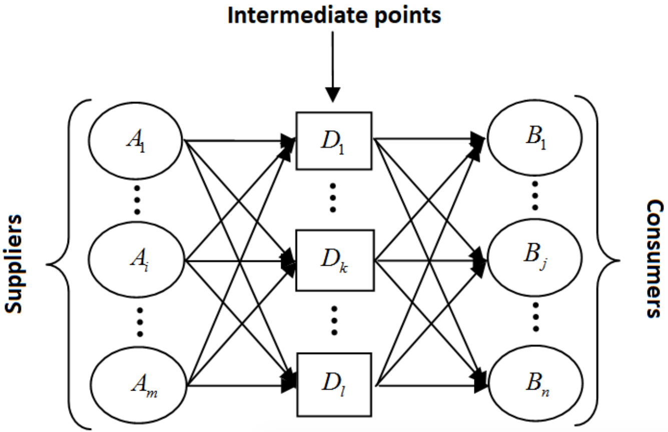
Connection system "A → D → B"



Formulation of the problem

- m supply points $A_1, \dots, A_m$ with $a_1, \dots, a_m$ product items
- n consumers $B_1, \dots, B_n$ satisfying their **unknown needs** with lower bounds $b_1^{low}, \dots, b_n^{low}$ and upper bounds $b_1^{up}, \dots, b_n^{up}$
- l intermediate points $D_1, \dots, D_l$ could be engaged

Optimal plan of product transportation is to be found

- c_{ik} is the cost of transportation of a product item from supplier A_i to intermediate point D_k
- c_{kj} is the cost of transportation of a product item from intermediate point D_k to consumer B_j

Variables

$x_{ik}, \quad i = \overline{1, m},\ k = \overline{1, l}$

the number of product items, which are transported from supplier A_i to intermediate point D_k

$y_{kj}, \quad k = \overline{1, l},\ j = \overline{1, n}$

the number of product items, which are transported from intermediate point D_k to consumer B_j

$z_j, \quad j = \overline{1, n}$

the number of product items, which are transported to consumer B_j

Formulation of the problem

$$f^*_{xyz} = \min_{x,y} \left\{ f(x,y,z) = \sum_{i=1}^{m} \sum_{k=1}^{l} c_{ik} x_{ik} + \sum_{k=1}^{l} \sum_{j=1}^{n} c_{kj} y_{kj} \right\} \quad (1)$$

$$\sum_{k=1}^{l} x_{ik} = a_i, \quad i = \overline{1,m}, \quad (2)$$

$$\sum_{k=1}^{l} y_{kj} = z_j, \quad j = \overline{1,n}, \quad (3)$$

$$\sum_{i=1}^{m} x_{ik} - \sum_{j=1}^{n} y_{kj} = 0, \quad k = \overline{1,l}, \quad (4)$$

$$\sum_{j=1}^{n} z_j = \sum_{i=1}^{m} a_i, \quad (5)$$

$$b_j^{low} \le z_j \le b_j^{up}, \quad j = \overline{1,n}, \quad (6)$$

$$x_{ik} \ge 0, \; y_{kj} \ge 0, \quad i = \overline{1,m}, k = \overline{1,l}, j = \overline{1,n}. \quad (7)$$

Problem (1)–(7) properties

- Problem (1)–(7) is a linear programming problem with $m \times l + n \times (l + 1)$ continuous variables x, y, z, and $m + 3n + l + 1$ linear constraints
- Assume $m \geq 2$, $n \geq 2$, $l \geq 2$, as well as $a_i > 0$, $b_j^{up} \geq b_j^{low} > 0$, for all $i = \overline{1, m}$, $j = \overline{1, n}$
- Problem (1)–(7) is balanced

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Consistency conditions 1

Theorem

Constraint system (2)–(7) is consistent if and only if the following condition is satisfied:

$$\sum_{j=1}^{n} b_j^{low} \leq \sum_{i=1}^{m} a_i \leq \sum_{j=1}^{n} b_j^{up}. \tag{8}$$

Remark. Inequality (8) means that constraints (2)–(5) are linearly dependent, and one arbitrary equality from (2) and (4) can be eliminated.

Consistency conditions 2

If $b_j^{low} = b_j^{up} = b_j$ for all $j = \overline{1, n}$, then the problem (1)–(7) turns into a classic balanced two-stage transportation problem, for which the following theorem holds.

Theorem

Constraint system of a classic balanced TTP is consistent if and only if the following condition is satisfied:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j. \tag{9}$$

Remark. The equality (9) means that constraints of classic balanced TTP are linearly dependent, and one arbitrary equality from them can be eliminated.

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Problem (1)–(7) with fixed number of consumers

$$f^*_{xyz} = \min_{x,y} \left\{ f(x,y,z) = \sum_{i=1}^{m} \sum_{k=1}^{l} c_{ik} x_{ik} + \sum_{k=1}^{l} \sum_{j=1}^{n} c_{kj} y_{kj} \right\} \qquad (10)$$

$$\sum_{k=1}^{l} x_{ik} = a_i, \ \sum_{k=1}^{l} y_{kj} = z_j, \quad i = \overline{1,m}, \ j = \overline{1,n}, \qquad (11)\text{-}(12)$$

$$\sum_{i=1}^{m} x_{ik} - \sum_{j=1}^{n} y_{kj} = 0, \quad k = \overline{1,l}, \qquad (13)$$

$$\sum_{j=1}^{n} z_j = \sum_{i=1}^{m} a_i, \qquad (14)$$

$$v_j b_j^{low} \leq z_j \leq b_j^{up} v_j, \quad j = \overline{1,n}, \qquad (15)$$

$$\sum_{j=1}^{n} v_j = d, \quad 1 < d < n, \qquad (16)$$

$$x_{ik} \geq 0, \ y_{kj} \geq 0, \ v_j = 0 \vee 1, \quad i = \overline{1,m}, k = \overline{1,l}, j = \overline{1,n}. \qquad (17)$$

Problem (10)–(17) properties

- Problem (10)–(17) is a Boolean linear programming problem with $m \times l + n \times (l + 1)$ continuous variables x, y, z, n Boolean variables v, and $m + 3n + l + 2$ linear constraints
- If $v_j = 0$, then products are not transported to consumer B_j; otherwise $v_j = 1$ and inequality $b_j^{low} \leq z_j \leq b_j^{up}$ holds
- Equality (16) means that products are transported to only d $(1 < d < n)$ chosen consumers

Conclusions

1. Necessary and sufficient conditions of consistency of constraints system of TTP with unknown consumer demands **are justified**

2. Modification of TTP with unknown consumer demands and fixed number of consumers **is considered**

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Questions?

Thank you for your attention

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