On Two-stage Transportation Problem

P. Stetsyuk, V. Stovba

V.M. Glushkov Institute of Cybernetics of the NAS of Ukraine

Mathematical Modeling,
Optimization and Information Technologies
(MMOIT-2021)

November 15-19, 2021

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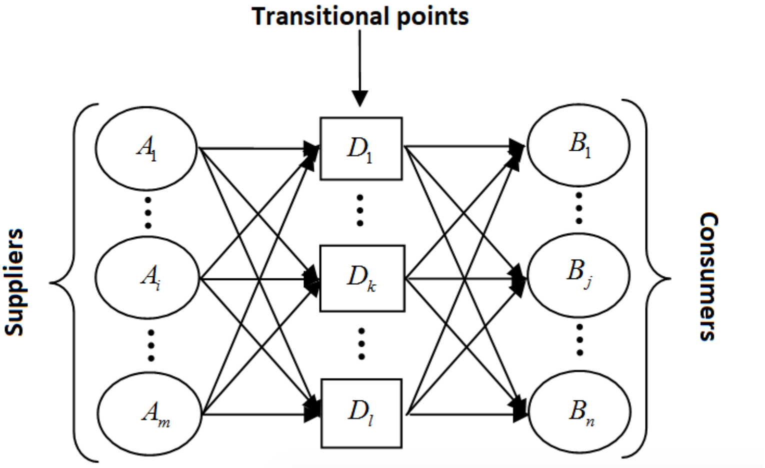
Connection system "A → D → B"



Formulation of the problem

Let there be m supply points $A_1, \dots, A_m$ with $a_1, \dots, a_m$ product items, which are to be transported to n consumers $B_1, \dots, B_n$, satisfying their needs $b_1, \dots, b_n$. To transport production from suppliers to consumers l transitional points $D_1, \dots, D_l$ could be engaged.

Optimal plan of product transportation is to be found, where c_{ik} – expenses for transportation of product item from supplier A_i to transitional point D_k, and c_{kj} – expenses for transportation of product item from supplier D_k to consumer B_j.

Variables

x_{ik} – the number of products, which are transported from supplier A_i to transitional point D_k

y_{kj} – the number of products, which are transported from transitional point D_k to consumer B_j

Formulation of the problem

$$f^{*} = \min_{x \geq 0, y \geq 0} \left\{ f(x,y) = \sum_{i=1}^{m} \sum_{k=1}^{l} c_{ik} x_{ik} + \sum_{k=1}^{l} \sum_{j=1}^{n} c_{kj} y_{kj} \right\} \qquad (1)$$

$$\sum_{k=1}^{l} x_{ik} = a_i, \quad i = 1, \ldots, m, \qquad (2)$$

$$\sum_{k=1}^{l} y_{kj} = b_j, \quad j = 1, \ldots, n, \qquad (3)$$

$$\sum_{i=1}^{m} x_{ik} = \sum_{j=1}^{n} y_{kj}, \quad k = 1, \ldots, l. \qquad (4)$$

Problem (1)–(4) is a linear programming problem with $(m+n) \times l$ variables та $m+n+l$ constraints

Objective function and constraints

Objective function (1) determines total cost for product transportation from suppliers to consumers through transitional points.

Constraints (2) determine nessecity of transportation of all products $a_1, \ldots, a_m$ from supply points to transitional points, and constraints (3) mean that products $b_1, \ldots, b_n$ must be transported from transitional points to consumers.

Constraints (4) determine conditions that all products that arrive from suppliers to each transitional point must be sent to consumers.

Consistency conditions

Lemma 1 (Karahodova et al.)

Constraints (2)–(4) are consistent, if the following condition is true:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j. \tag{1.1}$$

Karahodova O.O., Kigel V.R., Rozhok V.D. Operations Research: Textbook. – K.: Tsentr uchbovoi literatury, 2007. – 256 P.

Specialized algorithms

Karahodova O.O., Kigel V.R., Rozhok V.D. Operations Research: Textbook. – K.: Tsentr uchbovoi literatury, 2007. – 256 P.

Nakonechnyi S.I., Savina S.S. Mathematical Programming: Textbook. – K.: KNEU, 2003. – 452 P.

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Formulation of the problem

$$f^* = \min_{x\geq 0, y\geq 0} \left\{ f(x,y) = \sum_{i=1}^{m} \sum_{k=1}^{l} c_{ik} x_{ik} + \sum_{k=1}^{l} \sum_{j=1}^{n} c_{kj} y_{kj} \right\} \qquad (1)$$

subject to constraints (2)–(4) and

$$d_k^{low} \leq \sum_{i=1}^{m} x_{ik} \leq d_k^{up}, \quad k = 1, \dots, l, \qquad (5)$$

$d_1^{low}, \dots, d_l^{low}$ – minimal, and $d_1^{up}, \dots, d_l^{up}$ – maximal transitional points capacities $D_1, \dots, D_l$.

Problem (1)–(5) is a linear programming problem with $(m+n)\times l$ variables and $m+n+3l$ constraints.

Additional constraints

Constraints (5) determine lower and upper bounds for transitional points capacities.

They can be rewritten in the following way:

$$d_k^{low} \le \sum_{j=1}^{n} y_{kj} \le d_k^{up}, \quad k = 1, \dots, l. \qquad (5a)$$

Consistency conditions

Lemma 2 (Nakonechnyi, Savina)

Constraints (2)–(5) are consistent, if the following condition is true:

$$\sum_{k=1}^{l} d_k^{low} \leq \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \leq \sum_{k=1}^{l} d_k^{up}. \qquad (2.1)$$

Nakonechnyi S.I., Savina S.S. Mathematical Programming: Textbook. – K.: KNEU, 2003. – 452 P.

Specialized algorithms

Karahodova O.O., Kigel V.R., Rozhok V.D. Operations Research: Textbook. – K.: Tsentr uchbovoi literatury, 2007. – 256 P.

Nakonechnyi S.I., Savina S.S. Mathematical Programming: Textbook. – K.: KNEU, 2003. – 452 P.

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Additional data and variables

For product transportation I transitional points $D_1, \ldots, D_I$ are used with minimal $d_1^{low}, \ldots, d_I^{low}$ and maximal $d_1^{up}, \ldots, d_I^{up}$ capacities.

It is needed to

find an optimal plan of product transportation, which uses D $(1 < D < I)$ transitional points.

Let z_k be boolean variable, which equals one if transitional point D_k is used, and zero otherwise.

Formulation of the problem

$$f^* = \min_{x \geq 0, y \geq 0} \left\{ f(x,y) = \sum_{i=1}^{m} \sum_{k=1}^{l} c_{ik} x_{ik} + \sum_{k=1}^{l} \sum_{j=1}^{n} c_{kj} y_{kj} \right\} \quad (1)$$

subject to constraints (2)–(4) and

$$d_k^{low} z_k \leq \sum_{i=1}^{m} x_{ik} \leq d_k^{up} z_k, \quad k = 1, \ldots, l, \quad (5)$$

$$\sum_{k=1}^{l} z_k = D, \quad (6)$$

$$z_k = 0 \vee 1, \quad k = 1, \ldots, l. \quad (7)$$

Problem (1)–(7) is a boolean linear programming problem with $(m+n+1) \times l$ variables and $m+n+3l+1$ constraints.

Additional constraints

Constraints (6) mean that D transitional points are used, and constraints (5) determine lower and upper bounds for capacities of them.

Constraints (5) can be rewritten in the following way:

$$d_k^{low} z_k \leq \sum_{j=1}^{n} y_{kj} \leq d_k^{up} z_k, \quad k = 1, \ldots, l, \tag{5a}$$

Consistency conditions

Lemma 3

Constraints (2)–(7) are consistent, if the following condition is true:

$$D \times d^{low} \leq \sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \leq \sum_{k=1}^{D} d_k^{up}. \tag{3.1}$$

$d_1^{low} = d_2^{low} = \ldots = d_l^{low} = d^{low}$, and $d_1^{up}, \ldots, d_l^{up}$ are sorted in descending order $d_1^{up} \geq d_2^{up} \geq \ldots \geq d_l^{up}$.

AMPL-implementations

Stetsyuk P.I., Maziutynets G.V., Mileshovskyi B.I. AMPL-implementation of Two-stage Transportation Problem // Mathematical and Program Implementations of Intelligent Systems (MPZIS-2017), November 22–24, 2017. – D.: DNU, 2017. – P. 186–191.

Stetsyuk P.I., Mitza O.V., Streliuk O.V., Fesiuk O.V. Transportation Problem with Constraints on Transitional Point Capacities // Applied Mathematics and Mathematical Modeling Problems. – D.: DNU, 2017. – Vol. 17. – P. 207–219.

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Application 1

Problem (1)–(7) and its special cases are actual for agricultural enterprises for distribution and delivery of grown products for sale or processing using own capacities. Here transitional points are own or rented grain elevators.

Application 2

Problem (1)–(7) can be used for determination of rational location of given number of storages considering appointed locations of suppliers and consumers of material and technical means on the teritory, where their works are done (Trehubenko, 2015).

Romanchenko I.S., Khazanovych O.I., Trehubenko S.S. Modeling of Logistics System. – Lviv: HPSNGFA of Ukraine, 2015. – 156 P.

Work is supported by

CRDF Global
(grant G-202102-68020)

Questions?

Thank you for attention