Storage Location Problem: Properties and Computational Aspects

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Storage location problem (SLP)

- m storages with known capacities c_i $(i = \overline{1, m})$;
- n markets with known volumes r_j $(j = \overline{1, n})$;
- (x_i, y_i) – unknown coordinates of i-th storage;
- (a_j, b_j) – known coordinates of j-th market (consumer);
- $d_{ij} = \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2}$ – distance from i-th storage to j-th market;
- z_{ij} – volume of products, transported from i-th storage to j-th market.

The problem is to locate storages so that the total distance, calculated with weighting coefficients z_{ij} (volumes of products transported from storages to markets), is minimal.

Formulation of the problem

$$f(\mathrm{x},\mathrm{y},\mathrm{z}) = \sum_{i=1}^{m}\sum_{j=1}^{n} z_{ij}\sqrt{(x_i - a_j)^2 + (y_i - b_j)^2} \to \min \qquad (1)$$

$$\sum_{j=1}^{n} z_{ij} \le c_i, \quad i = \overline{1, m}, \qquad (2)$$

$$\sum_{i=1}^{m} z_{ij} = r_j, \quad j = \overline{1, n}, \qquad (3)$$

$$z_{ij} \ge 0, \quad i = \overline{1, m}, \ j = \overline{1, n}. \qquad (4)$$

The problem (1)–(4) is a multiextremal nonlinear programming problem. The function (1) is non-convex. If storage locations (x_i, y_i) are known, problem (1)–(4) is an opened transportation problem.

Properties of the objective function (1)

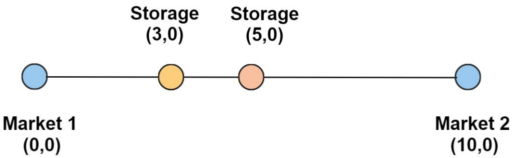
Lemma 1. For the function (1) and arbitrary $\lambda \in [0,1]$ the following inequality is fullfilled:

$$f(\lambda x^1 + (1-\lambda)x^2, \lambda y^1 + (1-\lambda)y^2, \lambda z^1 + (1-\lambda)z^2) \leq$$

$$\leq \sum_{i=1}^{m} \sum_{j=1}^{n} \lambda f(x^1, y^1, z^1) + (1-\lambda) f(x^2, y^2, z^2) + \lambda(\lambda - 1) \times \qquad (5)$$

$$\times \left(\sqrt{(x_i^1 - a_j)^2 + (y_i^1 - b_j)^2} - \sqrt{(x_i^2 - a_j)^2 + (y_i^2 - b_j)^2} \right) (z_{ij}^1 - z_{ij}^2).$$

Solution uniqueness of the problem (1)–(4)



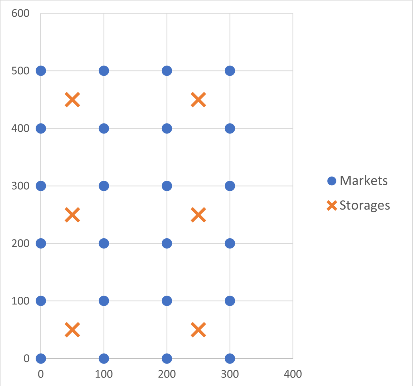
- The solution of the problem (1)–(4) is not unique (easy to see for $m = 1$ and $n = 2$)
- Removing square root from the function (1) makes it smooth and non-convex. In this case the problem (1)–(4) for $m = 1$ and $n = 2$ has a unique solution.

Consistency of constraint system (2)–(4)

Lemma 2. Constraint system (2)–(4) is consistent if and only if $\sum_{i=1}^{m} c_i \geq \sum_{j=1}^{n} r_j$.

Remark. If $\sum_{i=1}^{m} c_i = \sum_{j=1}^{n} r_j$, coefficient matrix of basis variables is non-degenerate, but it can contain zero coefficients for additional variables. Therefore, it is advisable to ensure that the condition $\sum_{i=1}^{m} c_i > \sum_{j=1}^{n} r_j$ is fullfilled.

Example 1: problem (1)–(4), $m = 6$, $n = 24$



- $c_i = 40 \ (i \in \overline{1,6})$, $r_j = 10 \ (j \in \overline{1,24})$
- Starting point (x_0, y_0, z_0) is obtained using presudorandom number generator

Example 1: MINOS 5.51 results

- **First** variant: $c_i = 40$; **second** variant: $c_i = 40.4$ $(i \in \overline{1,6})$
- MINOS 5.51 solver was used

	First variant	**Second variant**
Iterations number	368	163
Solving time (sec)	0.016978	0.004625
$\hat{\mathrm{f}}^*(\mathrm{x},\mathrm{y},\mathrm{z})$	18472.1	16970.6
Δ	0.088481	4.2874e-16

Here $\Delta = \left(\hat{f}^*(x,y,z) - f^*(x^*,y^*,z^*)\right)/f^*(x^*,y^*,z^*)$.

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Formulation of the problem

$$f(\mathbf{x}, \mathbf{y}, \mathbf{z}) = \sum_{i=1}^{m} \sum_{j=1}^{n} z_{ij} \sqrt{(x_i - a_j)^2 + (y_i - b_j)^2} \to \min \tag{6}$$

$$\sum_{j=1}^{n} z_{ij} = c_i, \quad i = \overline{1, m}, \tag{7}$$

$$\sum_{i=1}^{m} z_{ij} = r_j, \quad j = \overline{1, n}, \tag{8}$$

$$z_{ij} \geq 0, \quad i = \overline{1, m}, \; j = \overline{1, n}. \tag{9}$$

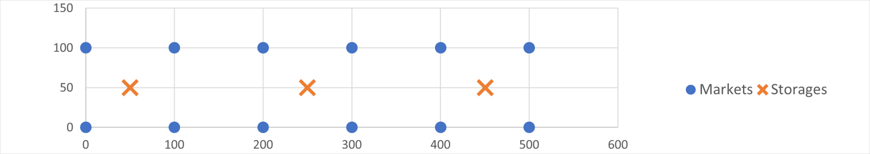
The problem (6)–(9) is a nonlinear programming problem.
Constraint system (7)–(9) is linearly dependent.

Degeneracy of the system (7)–(9)

Lemma 3. Constraint system (7)–(9) contains $m + n - 1$ linearly independent equations.

Degeneracy of the system (7)–(9) can significantly affect process of solving the problem (6)–(9) using simplex-type methods, so it is advisable to exclude one arbitrary linearly dependent constraint.

Example 2: problem (6)–(9), $m = 3$, $n = 12$



- $c_i = 40 \ (i \in \overline{1,3})$, $r_j = 10 \ (j \in \overline{1,12})$
- Starting point (x_0, y_0, z_0) is obtained using presudorandom number generator

	Problem A	Problem B	Problem C	Problem D
Active constraints	1,2,3	2,3	1,3	1,2

Example 2 results

MINOS 5.51 solver was used

	Problem A	Problem B	Problem C	Problem D
iter	47	46	33	41
obj	45	40	24	42
grad	44	39	23	41
Δ	8.36043e-15	2.1437e-16	1.02898e-14	5.14488e-15
time	0.00248	0.002263	0.002152	0.002174

SOLVED: Knitro, SNOPT, filter, Ipopt, LOQO
NOT SOLVED: CONOPT (D), LANCELOT (C)

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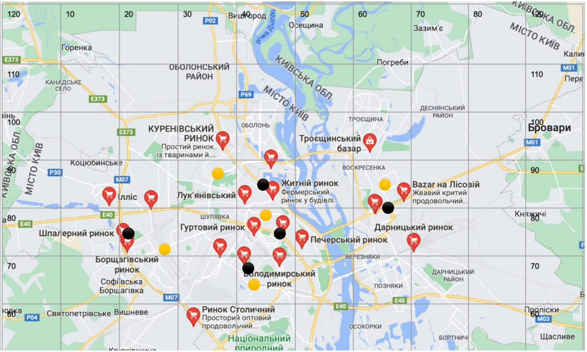
Example 3: problem (1)–(4), $m = 5$, $n = 19$



- $c_i = 40$ $(i \in \overline{1,5})$, $r_j = 10$ $(j \in \overline{1,19})$
- (x_0, y_0) are determined visully, z_0 is determined randomly
- Condition $\sum_{i=1}^{m} c_i > \sum_{j=1}^{n} r_j$ is fullfilled
- **filter** and **Knitro** solvers were used

Example 3: starting point dependence

- **5** starting points obtained using pseudorandom number generator, the **6th** point determined visually
- **filter** and **Knitro** solvers were used

St. p. No	filter		Knitro	
	$\hat{f}^*(x, y, z)$	iter	$\hat{f}^*(x, y, z)$	iter
1	1034.75	56	1015.96	86
2	1041.33	84	1277.8	39
3	1021.08	68	1016.33	78
4	1016.29	62	1016.38	71
5	1033.34	78	1017.07	125
6	**1015.9**	**58**	**1015.94**	**77**

Conclusions

Storage location problem is investigated

1. objective function is non-smooth and non-convex
2. constraint system consistency
3. degeneracy and linear dependence of constraint system

Conducted computational experiments

show how constraint system degeneracy and linear dependence affect NEOS solvers work depending on starting point

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Questions?

Thank you for attention