DENSE PACKING OF CIRCLES INTO A CIRCLE OF MINIMAL RADIUS

Problem formulation. Let a set of **N** circles with radii **r[i], i=1,2,…,N,** be given. Find centers **(x[i],y[i])**, **i=1,2,…,N**, of these circles and the radius **R** of an outer circle centered at **(0,0)** such that:

1. each of the circles **i=1,2,…,N** has to be completely inside the outer circle (any circle is allowed to touch the boundary of the outer circle),
2. for any pair **(i, j)** of circles with **i, j ∈ {1,...,N}** and **i < j**, the circles **i, j** do not overlap (touching is allowed),
3. the radius **R** of the outer circle is **as small as possible**.

Write a computer program that provides the centers **(x[i],y[i])**, **i=1,2,…,N**, and the radius **R**.

To evaluate your program**, ten** tests for **N=10, 20, 30, 40, 50** circles will be used.

For each of these 50 tests, points are given according to the following formula

$$round\left(\max\left\{0,\left(2-\frac{R}{R^*}\right)\right\}\times 100\right),$$

where **R** is the radius of the outer circle presented by the participant, $\mathbf{R}^*$ is the best (smallest) radius of the outer circle within the system.

Input data (which a participant can download for each of the 50 tests)

Line **1**: the number **N** of circles, where **N**∈{10,20,30,40,50}.

Lines **i=2,3,...,N+1**: the given radius **r[i-1]** of circle **i-1**.

The values of the radii are integers between 20 and 99.

Output data (which a participant should upload for each of the 50 downloaded tests)

Line **1**: the radius **R** determined by the program of the participant.

Lines **i=2,3,…,N+1** (three columns per line): the given radius **r[i-1]**, the coordinates **x[i-1]** and **y[i-1].**

The output results should be given up to four decimals after the dot.

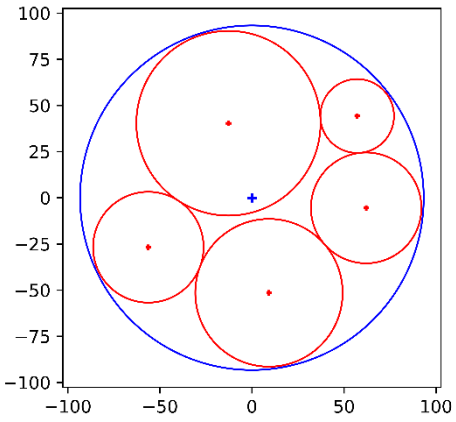
Example 1

Input data

```
5
20
30
30
40
50
```

Output data

```
92.2971
20  57.1656  44.2600
30 -56.2420 -26.7909
30  62.0537  -5.5006
40   9.2634 -51.4701
50 -12.7245  40.3376
```



N=5, r_1 =20, r_2 =30, r_3 =30, r_4 =40, r_5 =50

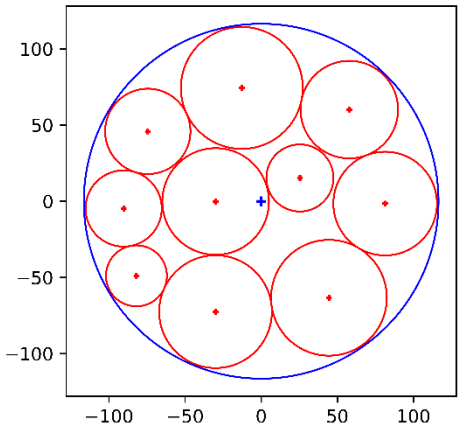
Example 2

Input data

```
10
20
22
25
28
32
34
35
37
38
40
```

Output data

```
115.4203
20 -81.8954 -48.9710
22  25.4276  15.2472
25 -90.2948  -4.7618
28 -74.4530  45.8154
32  57.8534  60.0993
34  81.4054  -1.5555
35 -29.9084  -0.1791
37 -29.9645 -72.4698
38  44.4811 -63.3666
40 -12.7245  74.3391
```



N=10, r_1 =20, r_2 =22, r_3 =25, r_4 =28, r_5 =32, r_6 =34, r_7 =35, r_8 =37, r_9 =38, r_{10} =40