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ПРИВЕДЕНИЕ ЗАДАЧИ ОПРЕДЕЛЕНИЯ ЗНАЧЕНИЙ И ОБЛАСТЕЙ ПОСТОЯНСТВА ФИЛЬТРАЦИОННЫХ ПАРАМЕТРОВ К ЗАДАЧЕ СЕТЕВОЙ СТРУКТУРЫ

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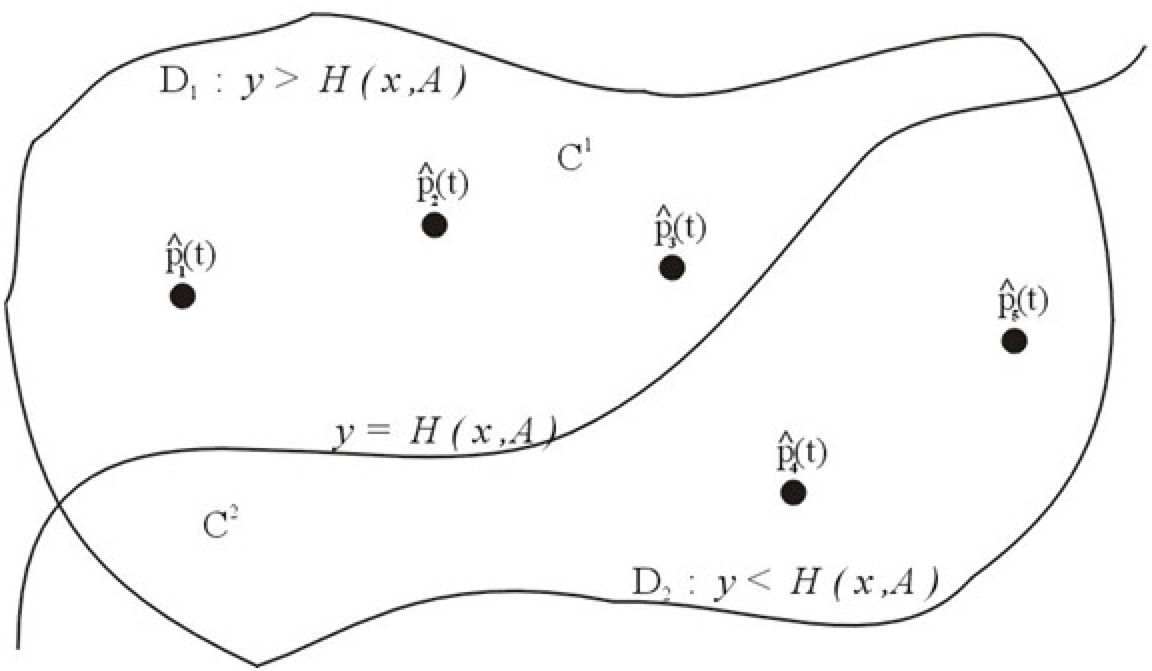
ABSTRACT

We consider the problem of parametric identification of the processes, described by parabolic equations by the example of underground oil filtration. The parameters being identified belong to the given classes of functions, for example, to piecewise linear and piecewise constant functions. The problem is not only to identify coefficient values, but also to determine the boundaries of coefficients constancy. For numerical solution of the problem we propose the same approach based on bringing original problem to finite optimization problem with special structure of constraints.



PROBLEM FORMULATION

We examine the process of two-dimensional oil filtration with constant viscosity. The goal is to identify piecewise constant values of permeability coefficient and its constancy boundaries. The considered oil filtration process in porous medium can be described by boundary problem for the parabolic equation of the following type:

$$b(x,y)\frac{\partial p}{\partial t} - div(a(x,y)grad\, p) + \sum_{l=1}^{N} q_l(t)\delta(x - x^l, y - y^l) = 0, (x,y) \in D, t \in (0,T], \quad (1)$$

$$p(x,y,0) = p_0(x,y), (x,y) \in D,$$

$$p(x,y,t)\big|_{(x,y)\in\Gamma_1} = \varphi(x,y,t), \quad \left.\frac{dp(x,y,t)}{dn}\right|_{(x,y)\in\Gamma_2} = \psi(x,y,t), \quad t \in (0,T], \quad (2)$$

$$a(x,y) = \sigma(x,y)H(x,y)/\mu, \;\; b(x,y) = H(m_0\beta_l + \beta_r).$$

Here $p = p(x,y,t)$ is a function, defining pressure at point $(x,y) \in D$ at time instance t; $D \subset E^2$ is filtration region; $\Gamma = \Gamma_1 \cup \Gamma_2$, $\Gamma_1 \cap \Gamma_2 = \varnothing$ is the boundary of the region D; $p_0(x,y), \varphi(x,y,t), \psi(x,y,t)$ are given smooth enough functions, defining initial edge conditions of the problem; $\sigma(x,y)$ is a permeability coefficient; $H(x,y)$ is a layer thickness at the point $(x,y) \in D$; μ is oil viscosity coefficient; m_0 is porosity coefficient; β_l, β_r are oil and porous medium compressibility coefficients; N is total number of wells; (x^l, y^l) and $q_l(t)$ are the coordinates and flow rate of l^{th} well, respectively, $q_l(t) > 0$ for producing and $q_l(t) < 0$ for injecting wells, $l = 1,\dots,N$; $\delta(\cdot,\cdot)$ is a two-dimensional Dirac function.

Let us suppose that domain the D is partitioned by curves $y = d^\tau(x, A^\tau), \tau = 1,\dots,s$ to non-overlapping regions $D_j, j = 1,\dots,\nu$, i.e.

PROBLEM FORMULATION

$$D = \bigcup_{j=1}^{\nu} D_j,\ D_j \cap D_i = \emptyset,\ j \neq i, D_j = \left\{(x,y) \in D : y - d^{\tau_i}\left(x, A^{\tau_i}\right) \leq 0, i = 1,\ldots,m_j\right\},$$
$$d^{\tau}\left(x, A^{\tau}\right) = \sum_{r=1}^{L} A_r^{\tau} \xi_r(x), \tau = 1,\ldots,s;\ i, j = 1,\ldots,\nu. \quad (3)$$

Permeability coefficients in each of the regions belong to given class of functions defined as follows:

$$\sigma(x,y) = \sigma_j(x,y), \qquad \sigma_j(x,y) = \sum_{i=1}^{K} c_i^j \chi_i(x,y),\ (x,y) \in D_j, \quad j = 1,\ldots,\nu, \quad (4)$$

Here $A = \left(A^1,\ldots,A^s\right) = \left(A_1^1,\ldots,A_L^1,\ldots,A_1^s,\ldots,A_L^s\right)$ and $C = \left(C^1,\ldots,C^{\nu}\right) = = \left(c_1^1,\ldots,c_K^1,\ldots,c_1^{\nu},\ldots,c_K^{\nu}\right)$; the functions $d^{\tau}\left(x, A^{\tau}\right)$ defining regions D_j have given form and depend on values of parameters A^{τ} being identified, $d^{\tau}\left(x, A^{\tau}\right)$ are smooth enough functions with corresponding inverse functions $x = \hat{d}^{\tau}\left(y, A^{\tau}\right)$, $j = 1,\ldots,\nu, \tau = 1,\ldots,s$; constant coefficients c_i^j, $i = 1,\ldots,K, j = 1,\ldots,\nu$ define permeability function $\sigma(x,y)$ in the region D_j, $j = 1,\ldots,\nu$; $\chi_i(x,y), i = 1,\ldots,K,\ \xi_r(x), r = 1,\ldots,L$ are given basis functions.

Let us suppose that on some wells or on all the wells observations of pressure values have been made:

$$\hat{p}_l(t) = p\left(x^l, y^l, t\right), \quad l \in Q \subseteq \{1,\ldots,N\},\ t \in [0,T], \quad (5)$$

here Q is the set of observed wells.

The goal is to determine such parameter values C, A from conditions (1)-(5) that calculated values of pressure are close to the actually observed ones. For that purpose let's introduce the following functional:

$$I(A,C;p) = \sum_{l \in Q} \int_0^T \left[p\left(x^l, y^l, t; C, A\right) - \hat{p}_l(t)\right]^2 dt + \varepsilon_1 \|A\|^2 + \varepsilon_2 \|C\|^2 \to \min_{C,A}, \quad (6)$$

where $p(x,y,t;C,A)$ is a function, defining pressure value from (1)-(4), corresponding to the parameters values C, A; $\varepsilon_1, \varepsilon_2 > 0$ are normalization parameters. **The identification problem (1)-(6) consists in determination of unknown parameters C, A and it belongs to the class of coefficient inverse problems.**

NUMERICAL SOLUTION OF DISCRETE PROBLEM

The formulated problem (1)-(6) is a problem of optimal parametric control of the system with distributed parameters, where vectors being optimized are:

$$A = \left(A_1^1,\dots,A_L^1,\dots,A_1^s,\dots,A_L^s\right) \in E^{Ls}, \quad C = \left(c_1^1,\dots,c_K^1,\dots,c_1^\nu,\dots,c_K^\nu\right) \in E^{K\nu}. \qquad (7)$$

with total dimension $(Ls + K\nu)$.

For numerical solution of the problem (1)–(6) we will apply some finite-difference approximation to the problem, we obtain a finite-dimensional problem of mathematical programming with equality and inequalities constraints of a special type. To solve this mathematical programming problem with the use of optimization methods of first order, we obtain the formulas of gradient in the space of coefficients, that determine the parameters of the process and the regions of constancy of coefficients C, A.

Without loss of generality of the proposed approach for numerical solving the problem, for sake of simplicity let us assume that $D = [0,a]\times[0,b]$, $\Gamma_2 = \emptyset$, and the coefficients participating in the permeability formula $\sigma(x,y)$ have two constancy regions:

$$D = D_1 \cup D_2, \qquad D_1 = \{(x,y): y - d(x,A) \ge 0\},$$
$$D_2 = \{(x,y): y - d(x,A) < 0\}, \ A = (A_1,\dots,A_L), \qquad \sigma(x,y) = \begin{cases} \sum_{i=1}^{K} c_i^1 \chi_i(x,y), & (x,y)\in D_1, \\ \sum_{i=1}^{K} c_i^2 \chi_i(x,y), & (x,y)\in D_2. \end{cases} \qquad (8)$$

Let us introduce a uniform grid in the domain $D \times [0,T]$

$$\Omega = \{(x_i, y_j, t_k) \mid x_i = ih_x,\ y_j = jh_y,\ t_k = kh_t,\ i = \overline{0,N_x},\ j = \overline{0,N_y},\ k = \overline{0,N_t},$$
$$h_x = a/N_x,\ h_y = b/N_y,\ h_t = T/N_t\}.$$

Let us introduce the following notations: $p_{ij}^k = p(x_i, y_j, t_k; A, C)$, $\sigma_{ij} = \sigma(x_i, y_j)$, $q_l^k = q_l(t_k)$, $b_{ij} = b(x_i, y_j)$, $H_{ij} = H(x_i, y_j)$.

NUMERICAL SOLUTION OF DISCRETE PROBLEM

We use integro-interpolational method in the vicinity of the line of discontinuity of $\sigma(x,y): y = d(x,A)$ and in the vicinity of the wells. After approximation with the use of implicit scheme with respect to t the boundary problem (1), (2), (8) and after some other manipulations we will obtain:

$$p_{ij}^{k+1} = \frac{b_{ij}h_x^2h_y^2}{b_{ij}h_x^2h_y^2 + h_t\left(h_y^2\left(a_{i+1,j}^x + a_{ij}^x\right) + h_x^2\left(a_{i,j+1}^y + a_{ij}^y\right)\right)}\left[p_{ij}^k + \right.$$
$$\left. + \frac{h_y^2\left(a_{i+1,j}^x p_{i+1,j}^{k+1} + a_{ij}^x p_{i-1,j}^{k+1}\right) + h_x^2\left(a_{i,j+1}^y p_{i,j+1}^{k+1} + a_{ij}^y p_{i,j-1}^{k+1}\right)}{b_{ij}h_x^2h_y^2} h_t - \frac{q_{ij}^{k+1}}{b_{ij}} h_t \right], \tag{9}$$

$$p_{0j}^{k+1} = \varphi_{0j}^{k+1},\ p_{N_xj}^{k+1} = \varphi_{N_xj}^{k+1},\ p_{i0}^{k+1} = \varphi_{i0}^{k+1},\ p_{iN_y}^{k+1} = \varphi_{iN_y}^{k+1},\ p_{ij}^0 = p_{0ij}, \tag{10}$$

$$a_{ij}^x = \begin{cases} \sigma_{i-1,j}H_{i-1,j}/\mu, & x_i < \hat{d}(y_j, A), \\ \left(\sigma_{i-1,j}H_{i-1,j}\left(\hat{d}(y_j,A) - x_{i-1}\right) + \sigma_{ij}H_{ij}\left(x_i - \hat{d}(y_j,A)\right)\right)/(\mu h_x), & x_{i-1} < \hat{d}(y_j,A) < x_i, \\ \sigma_{ij}H_{ij}/\mu, & x_{i-1} > \hat{d}(y_j,A), \end{cases} \tag{11}$$

$$a_{ij}^y = \begin{cases} \sigma_{i,j-1}H_{i,j-1}/\mu, & y_j < d(x_i, A), \\ \left(\sigma_{i,j-1}H_{i,j-1}\left(d(x_i,A) - y_{j-1}\right) + \sigma_{ij}H_{ij}\left(y_j - d(x_i,A)\right)\right)/(\mu h_y), & y_{j-1} < d(x_i,A) < y_j, \\ \sigma_{ij}H_{ij}/\mu, & y_{j-1} > d(x_i,A), \end{cases} \tag{12}$$

$$q_{ij}^{k+1} = \begin{cases} \sum\limits_{l \in M_{ij}} q_l^{k+1} \dfrac{\left(h_x - \left|x_i - x^l\right|\right)\left(h_y - \left|y_j - y^l\right|\right)}{h_x^2h_y^2}, & M_{ij} \neq \emptyset, \\ 0, & M_{ij} = \emptyset, \end{cases} \tag{13}$$

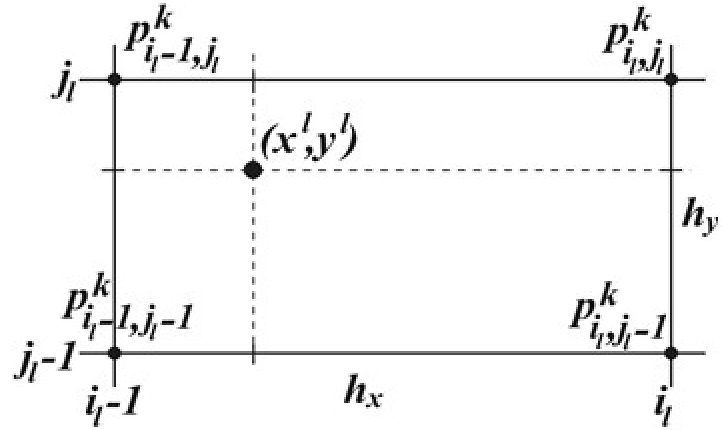
$$M_{ij} = \left\{l \in \{1,\ldots,N\} : \left(x^l, y^l\right) \in [x_{i-1}, x_{i+1}] \times [y_{j-1}, y_{j+1}]\right\},\ i = \overline{1, N_x - 1},\ j = \overline{1, N_y - 1},\ k = \overline{0, N_t - 1}.$$

NUMERICAL SOLUTION OF DISCRETE PROBLEM

Let us assume that l^{th} well belongs to $(i_l-1, j_l-1)^{\text{th}}$ cell of grid region, i.e. its coordinates satisfy inequalities $x_{i_l-1} \le x^l \le x_{i_l}$, $y_{j_l-1} \le y^l \le y_{j_l}$. Then value of well pressure can be approximated in the following way:

$$\begin{aligned}\bar{p}_l^k = p\left(x^l, y^l, t_k; A, C\right) = \Big(p_{i_l-1,j_l-1}^k\left(x_{i_l}-x^l\right)\left(y_{j_l}-y^l\right)+\\ + p_{i_l-1,j_l}^k\left(x_{i_l}-x^l\right)\left(y^l-y_{j_l-1}\right)+p_{i_l,j_l-1}^k\left(x^l-x_{i_l-1}\right)\left(y_{j_l}-y^l\right)+\\ + p_{i_l,j_l}^k\left(x^l-x_{i_l-1}\right)\left(y^l-y_{j_l-1}\right)\Big)\Big/\left(h_x h_y\right).\end{aligned} \tag{14}$$

Thus, the pressure $p = p\left(x^l, y^l, t_k; C, A\right)$ at the location point of the l^{th} well is defined by the values of pressure in the vertices of grid rectangle that encloses the l^{th} well. Substituting expression (14) into (6) and approximating integral, we will get the following expression for the functional:



$$I(A,C;p) = h_t \sum_{k=1}^{N_t} \sum_{l \in Q} \left[\bar{p}_l^k - \hat{p}_l^k\right]^2 + \varepsilon_1\|A\| + \varepsilon_2\|C\|, \tag{15}$$

that depends only on grid values of pressure function.

To find solution for mathematical programming problem (9)-(15), i.e. to determine parameters A and C, we use finite optimization methods, in particular gradient method:

$$A^{s+1} = A^s - \alpha_s \nabla_A I\left(A^s, C^s; p\right), \; C^{s+1} = C^s - \alpha_s \nabla_C I\left(A^s, C^s; p\right), \alpha_s > 0, \; s = 0,1,\ldots, \tag{16}$$

where A^0 and C^0 are given initial approximations; α_s, $s = 0,1,\ldots,$ is a step of one-dimensional minimization. In order to apply procedure (16), it is important to obtain formulas for components of functional (15) gradient with respect to parameters (7) being optimized. For that purpose we will derive formulas for

$$\nabla_A I(A,C;p) = \left(dI/dA_1, \ldots, dI/dA_L\right), \nabla_C I(A,C;p) = \left(dI/dc_1^1, \ldots, dI/dc_K^1, dI/dc_1^2, \ldots, dI/dc_K^2\right)$$

where dI/dA_i and dI/dc_i^j is a total partial derivative of function (15) in view of dependency of pressure on parameters C, A, that follows from relations (9), (10).

NUMERICAL SOLUTION OF DISCRETE PROBLEM

In order to determine total partial derivatives dI/dA and dI/dC, let us introduce a matrix of impulses [K. R. Aida-zade, *Zh. Vychisl. Matem. i Matemat. Fiziki* **29**, 3 (1989), 346-354]

$$V = \left(\left(V_{ij}^{\,k}\right)\right)_{i=\overline{0,N_x},\ j=\overline{0,N_y}}^{k=\overline{0,N_t}} = \left(\left(dI \,/\, dp_{ij}^{\,k}\right)\right)_{i=\overline{0,N_x},\ j=\overline{0,N_y}}^{k=\overline{0,N_t}}.$$

Here the derivatives are assumed as total in view of mutual relationship of pressure grid values from (9). So we have the following equations:

$$\begin{aligned} V_{ij}^{k} = {} & \frac{\partial I}{\partial p_{ij}^{k}} + \frac{\partial p_{i-1,j}^{k}}{\partial p_{ij}^{k}} V_{i-1,j}^{k} + \frac{\partial p_{i+1,j}^{k}}{\partial p_{ij}^{k}} V_{i+1,j}^{k} + \frac{\partial p_{i,j-1}^{k}}{\partial p_{ij}^{k}} V_{i,j-1}^{k} + \frac{\partial p_{i,j+1}^{k}}{\partial p_{ij}^{k}} V_{i,j+1}^{k} + \\ & + \frac{\partial p_{ij}^{k+1}}{\partial p_{ij}^{k}} V_{ij}^{k+1}, \quad i=\overline{1,N_x-1},\ j=\overline{1,N_y-1},\ k=\overline{0,N_t-1}, \end{aligned} \tag{17}$$

$$\begin{aligned} V_{ij}^{N_t} = {} & \frac{\partial I}{\partial p_{ij}^{N_t}} + \frac{\partial p_{i-1,j}^{N_t}}{\partial p_{ij}^{N_t}} V_{i-1,j}^{N_t} + \frac{\partial p_{i+1,j}^{N_t}}{\partial p_{ij}^{N_t}} V_{i+1,j}^{N_t} + \frac{\partial p_{i,j-1}^{N_t}}{\partial p_{ij}^{N_t}} V_{i,j-1}^{N_t} + \\ & + \frac{\partial p_{i,j+1}^{N_t}}{\partial p_{ij}^{N_t}} V_{i,j+1}^{N_t}, \quad i=\overline{1,N_x-1},\ j=\overline{1,N_y-1}, \end{aligned} \tag{18}$$

$$V_{i0}^{k} = V_{iN_y}^{k} = V_{0j}^{k} = V_{N_x j}^{k} = 0, \quad i=\overline{0,N_x},\ j=\overline{0,N_y},\ k=\overline{0,N_t}. \tag{19}$$

Let's call the system (17)-(19) an adjoint system to the system (9).

NUMERICAL SOLUTION OF DISCRETE PROBLEM

Partial derivatives in (17)-(19) are easily determined from (9) by direct differentiation. In particular, for $\partial p_{ij}^{k+1} / \partial p_{ij}^{k}$, $\partial p_{i-1,j}^{k} / \partial p_{ij}^{k}$, $\partial p_{i+1,j}^{k} / \partial p_{ij}^{k}$, $\partial p_{i,j-1}^{k} / \partial p_{ij}^{k}$, $\partial p_{i,j+1}^{k} / \partial p_{ij}^{k}$ we obtain:

$$\frac{\partial p_{ij}^{k+1}}{\partial p_{ij}^{k}} = \frac{b_{ij} h_x^2 h_y^2}{b_{ij} h_x^2 h_y^2 + h_t\left(h_y^2\left(a_{i+1,j}^{x} + a_{ij}^{x}\right) + h_x^2\left(a_{i,j+1}^{y} + a_{ij}^{y}\right)\right)},$$
$$\frac{\partial p_{i-1,j}^{k}}{\partial p_{ij}^{k}} = \frac{h_t h_y^2 a_{ij}^{x}}{b_{i-1,j} h_x^2 h_y^2 + h_t\left(h_y^2\left(a_{ij}^{x} + a_{i-1,j}^{x}\right) + h_x^2\left(a_{i-1,j+1}^{y} + a_{i-1,j}^{y}\right)\right)},$$
$$\frac{\partial p_{i+1,j}^{k}}{\partial p_{ij}^{k}} = \frac{h_t h_y^2 a_{i+1,j}^{x}}{b_{i+1,j} h_x^2 h_y^2 + h_t\left(h_y^2\left(a_{i+2,j}^{x} + a_{i+1,j}^{x}\right) + h_x^2\left(a_{i+1,j+1}^{y} + a_{i+1,j}^{y}\right)\right)}, \quad (20)$$
$$\frac{\partial p_{i,j-1}^{k}}{\partial p_{ij}^{k}} = \frac{h_t h_x^2 a_{ij}^{y}}{b_{i,j-1} h_x^2 h_y^2 + h_t\left(h_y^2\left(a_{i+1,j-1}^{x} + a_{i,j-1}^{x}\right) + h_x^2\left(a_{ij}^{y} + a_{i,j-1}^{y}\right)\right)},$$
$$\frac{\partial p_{i,j+1}^{k}}{\partial p_{ij}^{k}} = \frac{h_t h_x^2 a_{i,j+1}^{y}}{b_{i,j+1} h_x^2 h_y^2 + h_t\left(h_y^2\left(a_{i+1,j+1}^{x} + a_{i,j+1}^{x}\right) + h_x^2\left(a_{i,j+2}^{y} + a_{i,j+1}^{y}\right)\right)}.$$

Partial derivatives $\partial I / \partial p_{ij}^{k}$ are determined from relation (15):

$$\frac{\partial I}{\partial p_{ij}^{k}} = \begin{cases} h_t \sum\limits_{l \in Q \cap M_{ij}} 2\left[\bar{p}_l^k - \hat{p}_l^k\right] \dfrac{\partial \bar{p}_l^k}{\partial p_{ij}^{k}} = h_t \sum\limits_{l \in Q \cap M_{ij}} 2\left[\bar{p}_l^k - \hat{p}_l^k\right] \dfrac{\left|x^l - x_i\right| \cdot \left|y^l - y_j\right|}{h_x h_y}, & Q \cap M_{ij} \neq \emptyset, \\ 0, & Q \cap M_{ij} = \emptyset. \end{cases} \quad (21)$$

NUMERICAL SOLUTION OF DISCRETE PROBLEM

As a result we obtain a set of algebraic equations (17)-(19) with respect to V_{ij}^k, that we call an adjoint set to the system (9)-(10). Solving (17)-(19) we determine dI/dp_{ij}^k. Then gradient components $\nabla I = (dI/dA, dI/dC)$ are determined as follows:

$$\frac{dI}{dC^r} = \sum_{k=0}^{N_t} \sum_{(i,j)\in S_r} V_{ij}^k \frac{\partial p_{ij}^k}{\partial C^r} + \frac{\partial I}{\partial C^r}, \tag{22}$$

here $C = (C^1, C^2) = (c_1^1, c_2^1, \dots, c_K^1; c_1^2, c_2^2, \dots, c_K^2)$, $S_r = \{(i,j): D_r \cap [x_{i-1}, x_{i+1}] \times [y_{j-1}, y_{j+1}] \neq \varnothing\}$, $r = 1,2$.

The derivatives $\partial p_{ij}^k / \partial C^r$ are determined from relation (9) taking into account dependence of coefficients $a_{i+1,j}^x$, a_{ij}^x, $a_{i,j+1}^y$, a_{ij}^y on C (this dependence is determined by relations (11), (12)).

$$\frac{dI}{dA} = \sum_{k=0}^{N_t} \sum_{(\nu,\mu)\in\hat\Omega} \frac{\partial p_{\nu\mu}^k}{\partial A} V_{\nu\mu}^k + \frac{\partial I}{\partial A} = \sum_{k=0}^{N_t} \sum_{(\nu,\mu)\in\hat\Omega} V_{\nu\mu}^k \left(\frac{\partial p_{\nu\mu}^k}{\partial a_{\nu+1,\mu}^x} \frac{\partial a_{\nu+1,\mu}^x}{\partial A} + \frac{\partial p_{\nu\mu}^k}{\partial a_{\nu\mu}^x} \frac{\partial a_{\nu\mu}^x}{\partial A} + \frac{\partial p_{\nu\mu}^k}{\partial a_{\nu,\mu+1}^y} \frac{\partial a_{\nu,\mu+1}^y}{\partial A} + \frac{\partial p_{\nu\mu}^k}{\partial a_{\nu\mu}^y} \frac{\partial a_{\nu\mu}^y}{\partial A} \right) + \frac{\partial I}{\partial A}. \tag{23}$$

Here index set $\hat\Omega = \{(i,j): |y_j - d(x_i, A)| < h_y \text{ or } |x_i - \hat d(y_j, A)| < h_x\}$ includes grid points, which situate in the vicinity of the coefficient constancy region boundaries. At these points pressure values directly depend on vector A due to dependency of coefficients a_{ij}^x, a_{ij}^y on vector A in the view of (11), (12). In particular, from (11), (12) it is easy to obtain for $(i,j) \in \hat\Omega$ the following formulas:

$$\frac{\partial a_{ij}^y}{\partial A} = \frac{\sigma_{i,j-1} H_{i,j-1} - \sigma_{ij} H_{ij}}{\mu h_y} \cdot \frac{\partial d(x_i, A)}{\partial A}, \quad \frac{\partial a_{ij}^x}{\partial A} = \frac{\sigma_{i-1,j} H_{i-1,j} - \sigma_{ij} H_{ij}}{\mu h_x} \cdot \frac{\partial \hat d(y_j, A)}{\partial A}, \tag{24}$$

and derivatives $\partial p_{\nu\mu}^k / \partial a_{\nu+1,\mu}^x$, $\partial p_{\nu\mu}^k / \partial a_{\nu\mu}^x$, $\partial p_{\nu\mu}^k / \partial a_{\nu,\mu+1}^y$, $\partial p_{\nu\mu}^k / \partial a_{\nu\mu}^y$ are easily obtained from (9).

NUMERICAL SOLUTION OF DISCRETE PROBLEM

So, we have proved the following theorem.

Theorem. ***The components of functional gradient of discretized optimal control problem (9)-(15) with respect to optimized parameters*** $(A, C) = \left(A_1, \ldots, A_L; c_1^1, \ldots, c_K^1, c_1^2, \ldots, c_K^2\right)$ ***are defined by (22), (23) with account for solution of adjoint problem (17)-(19) and relations (20), (21), (24).***

Let us give the algorithm of numerical solving finite-dimensional minimization problem which is obtained after discretization of initial parametric identification problem (1)-(6).

Each iteration of the process (16) at current values of coefficients A^s and C^s, $s = 0, 1, \ldots$ consists in execution of the following steps:

STEP 1. For given values A^s and C^s we solve boundary problem (9), (10) and determine pressure values p_{ijs}^k, $i = \overline{1, N_x - 1}$, $j = \overline{1, N_y - 1}$, $k = \overline{1, N_t}$.

STEP 2. Using the values p_{ijs}^k, $i = \overline{1, N_x - 1}$, $j = \overline{1, N_y - 1}$, $k = \overline{1, N_t}$, and (17)-(19), we determine impulses values $V_s = \left(\left(V_{ijs}^k\right)\right)_{i = \overline{0, N_x},\, j = \overline{0, N_y}}^{k = \overline{0, N_t}}$.

STEP 3. Using found values of pressure p_{ijs}^k, $i = \overline{1, N_x - 1}$, $j = \overline{1, N_y - 1}$, $k = \overline{1, N_t}$ and of impulses $V_s = \left(\left(V_{ijs}^k\right)\right)_{i = \overline{0, N_x},\, j = \overline{0, N_y}}^{k = \overline{0, N_t}}$ from formulas (22), (23), we calculate gradient components $\nabla I = \left(dI/dA^s, dI/dC^s\right)$.

STEP 4. The procedure (16) to find α_s from one-dimension minimization of functional (15) is performed and new iterative values for A^{s+1}, C^{s+1} are determined.

STEP 5. If stop condition of iteration process is not met then we put $s = s + 1$ and go to *STEP 1,* otherwise iteration process is halted.

NUMERICAL EXPERIMENTS

Numerical results given below were obtained under the following conditions and at the following values of oil layer parameters :

$$D \times [0,T] = \{(x,y): 0 \le x \le 1000(m), 0 \le y \le 1000(m)\} \times [0;480(h)],$$

$$H(x,y) = const = 5\ (m),\quad p(x,y,0) = 0,\ (x,y) \in D,$$

$$p(x,y,t)\big|_{\Gamma_1} = 0, \qquad \frac{dp(x,y,t)}{dn}\bigg|_{\Gamma_2} = 0,\quad t \in (0,T],$$

$$m_0 = 0.2,\ \beta_l = 0.000225,\ \beta_r = 0.00001,\ \mu = 2.5\ (cP)$$

Let us suppose that 9 wells are placed in region D and that observation is carried out at all these wells (see fig. 1, the wells are marked by the symbol «•»). In the table 1, the coordinates and flow rates of the wells are given. All the given below results of numerical experimens are obtained by double precision calculations.

Table 1

№	*Well coordinates*	$q(m^3/h)$
1	(410;160)	7.6
2	(160;410)	8
3	(815;245)	8.4
4	(245;815)	9
5	(620;830)	10
6	(830;620)	10.6
7	(220;310)	11.3
8	(885;810)	12.2
9	(535;490)	11

NUMERICAL EXPERIMENTS

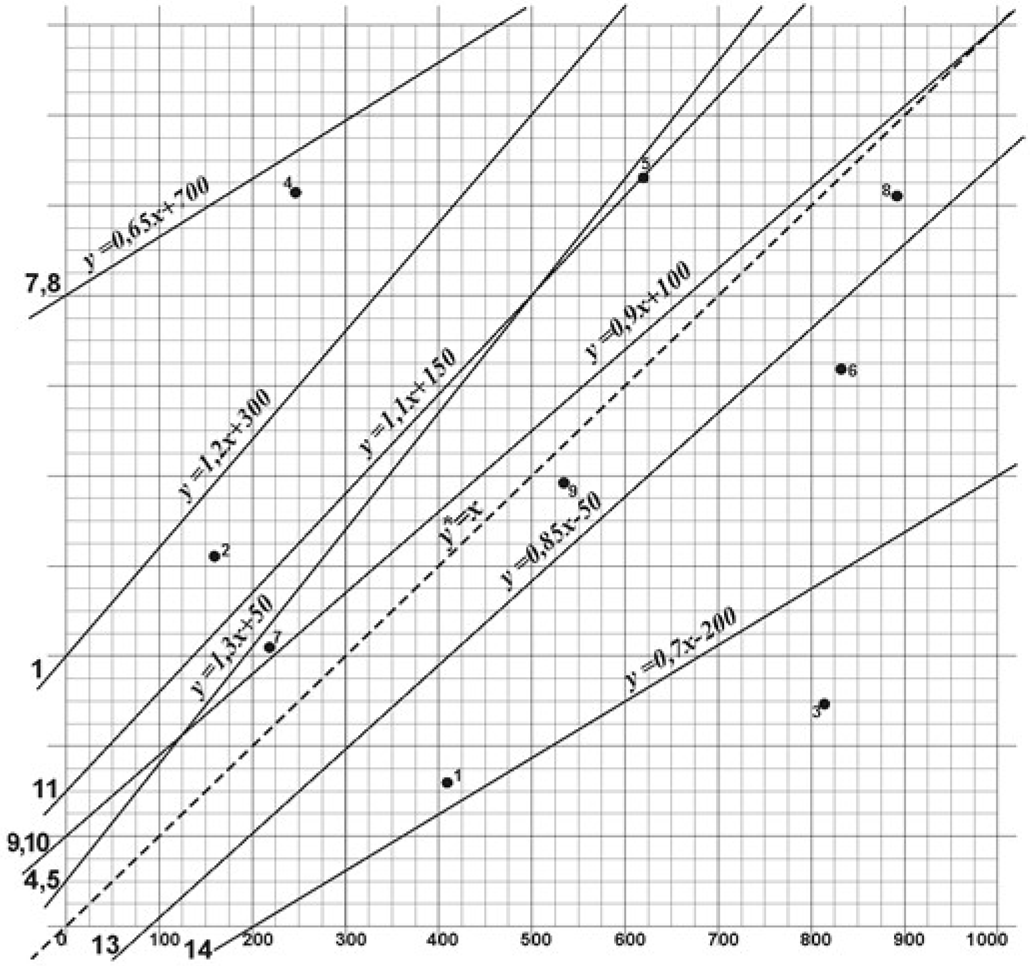


Figure 1. The scheme of wells location and of optimal and initial lines placement.

● – observable wells

_ _ _ _ – optimal line

_____ – initial lines

NUMERICAL EXPERIMENTS

Without loss of generality and for the sake of simplicity, let us suppose that permeability factor is a piecewise constant function and that the region D is divided into two subregions by a straight line which is described by equation: $y = A_1 x + A_2$.

$$a(x,y) = \begin{cases} \sigma_1 H/\mu, & (x,y) \in D_1, \\ \sigma_1 H/\mu, & (x,y) \in D_1, \end{cases}$$

$$D_1 = \{(x,y) : y - (A_1 x + A_2) \geq 0\}, D_2 = \{(x,y) : y - (A_1 x + A_2) < 0\}.$$

The region D is partitioned by straight line $y - x = 0$ with $\sigma_1 = 0.2\,(darcy);\ \sigma_2 = 0.5\,(darcy)$, i. e. $(A^*, \sigma^*) = (A_1^*, A_2^*, \sigma_1^*, \sigma_2^*) = (1;0;0.2;0.5)$. At this point the optimal value of functional is zero: $I(A^*, \sigma^*, P) = 0$. The analysis shows that objective functional is a steep ravine function (figure 2, 3). So, certain difficulties arise when solving the identification problem by first order optimization methods.

At the figure 1, we show location of wells, the position of optimal straight line that divides the region (shown as dashed line) and the positions of straight lines that correspond to various initial values of parameters used at optimization (shown as solid lines).

NUMERICAL EXPERIMENTS

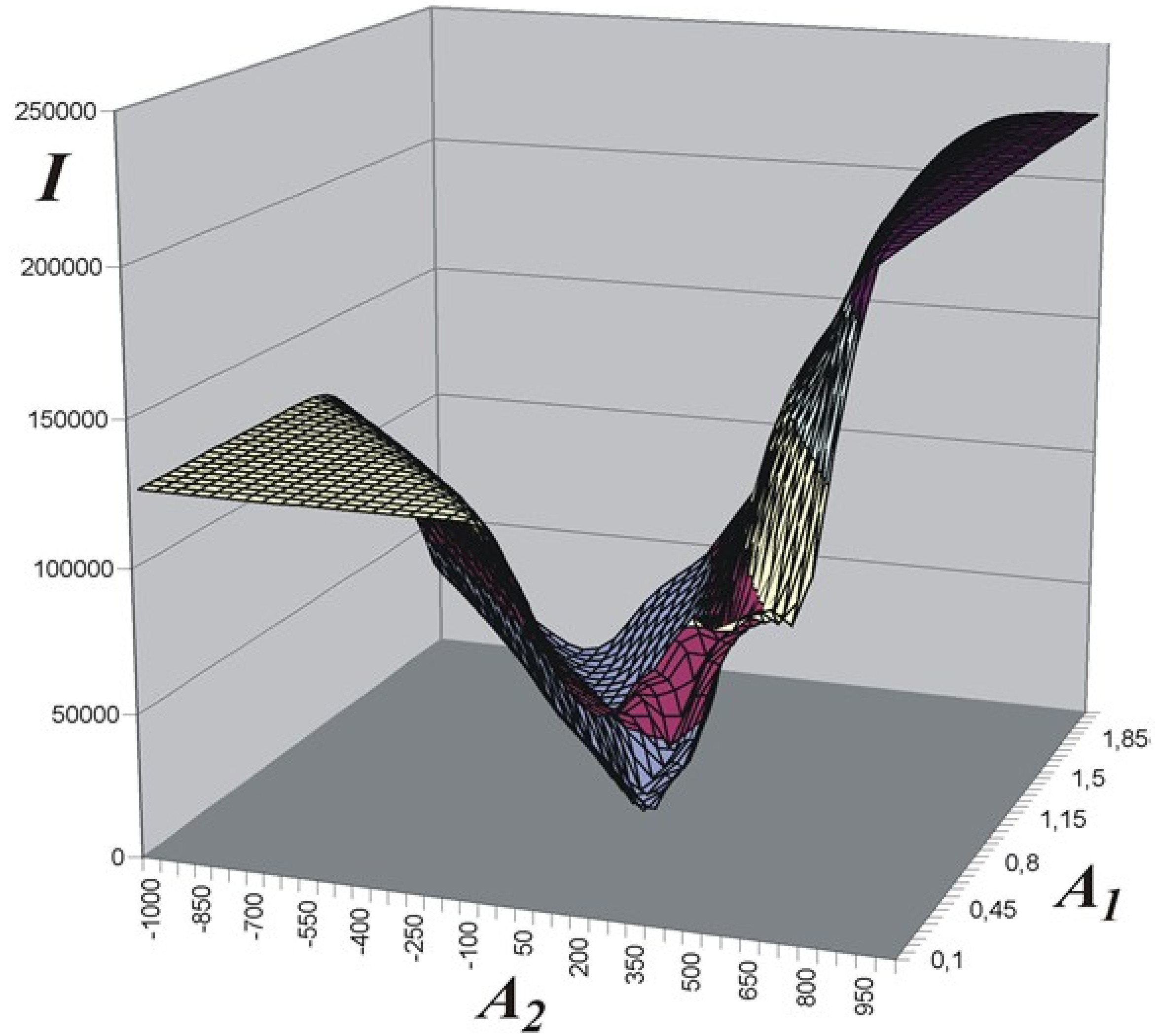


Figure 2. Graph of functional for $A_1 \in [0.1;2.1]$, $A_2 \in [-1000;1000]$ at fixed values of $\sigma_1 = 0.2$, $\sigma_2 = 0.5$.

NUMERICAL EXPERIMENTS

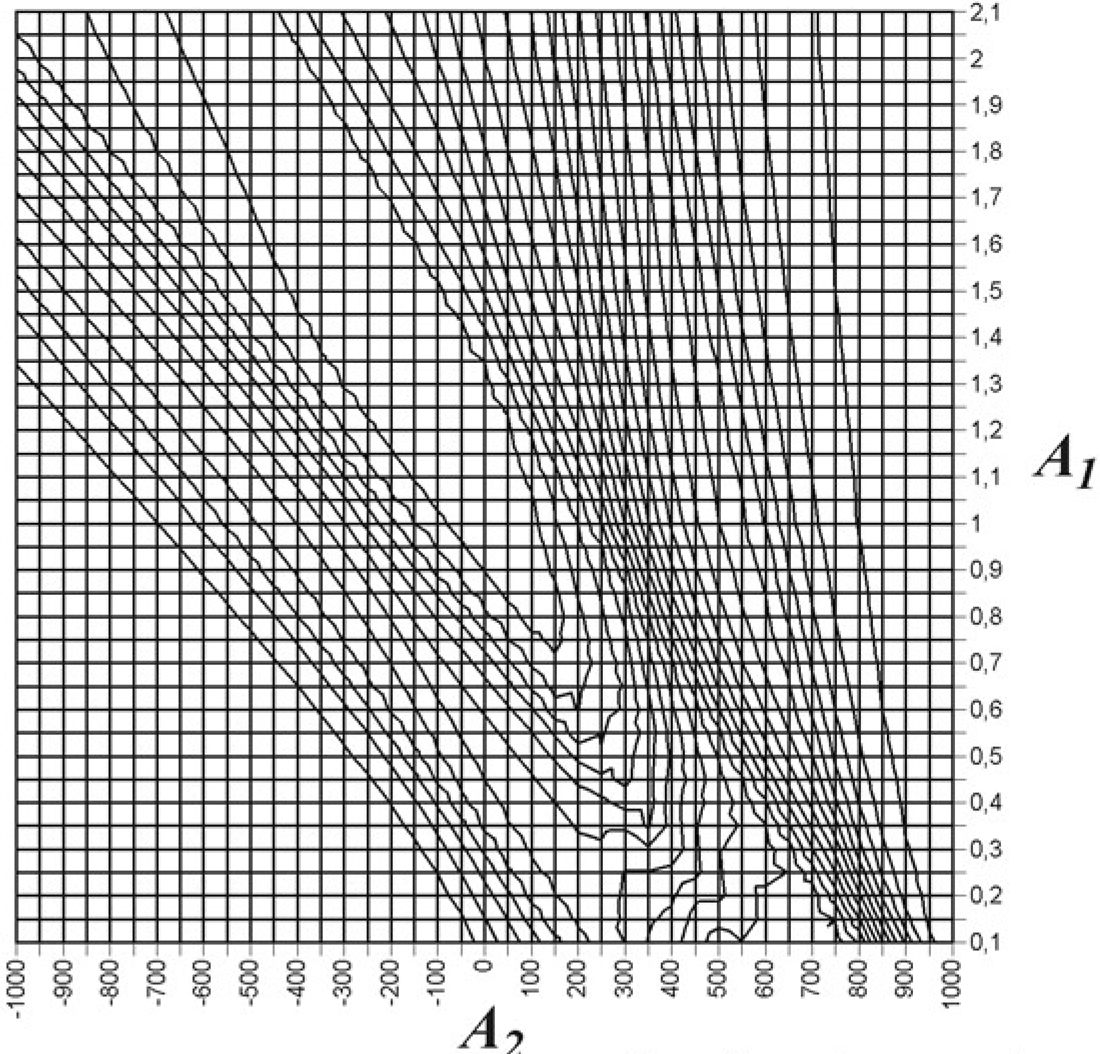


Figure 3. Level lines of functional for $A_1 \in [0.1;2.1]$, $A_2 \in [-1000;1000]$ at fixed values of $\sigma_1 = 0.2$, $\sigma_2 = 0.5$.

NUMERICAL EXPERIMENTS

To numerically solve the problem we have used conjugate gradient method in the space of (A,σ) parameters vector being identified. Numerical experiments were carried out at various initial values of (A^0,σ^0) and with steps $h_x = h_y = 25\,(m),\ h_t = 20\,(h)$.

Numerical results of solving parametric identification problem obtained at various values of vector (A,σ) being identified are presented at the table 2. Obtained approximate values of $(A^*_{app},\sigma^*_{app})$, the value of $I(A^*_{app},\sigma^*_{app};p)$ and the number of conjugate gradient method iterations are shown.

Table 2

№	(A^0,σ^0)	$I(A^0,\sigma^0;p)$	$(A^*_{app},\sigma^*_{app})$	$I(A^*_{app},\sigma^*_{app};p)$	*Iter. count*
1	(1.200;300.000;0.250;0.550)	33261.22916	(0.99978;0.4465;0.20001;0.50012)	0.01019	42
2	(1.200;50.000;0.250;0.600)	10019.63248	(0.99992;0.2961;0.20002;0.50013)	0.00689	40
3	(1.200;50.000;0.100;0.400)	567426.53869	(0.99995;0.0372;0.20001;0.49995)	0.00139	42
4	(1.300;50.000;0.250;0.600)	8766.29521	(0.99874;1.0193;0.20001;0.50007)	0.03025	49
5	(1.300;50.000;0.350;0.650)	62961.99465	(1.00344;-1.5188;0.20009;0.49992)	0.07397	40
6	(0.650;175.000;0.350;0.400)	51497.49194	(1.00056;-0.1459;0.20002;0.50008)	0.00628	43
7	(0.650;700.000;0.250;0.400)	67074.61886	(0.99982;0.1535;0.19999;0.50003)	0.00059	46
8	(0.650;700.000;0.150;0.450)	609981.00843	(0.99969;0.2618;0.20001;0.50004)	0.00166	32
9	(0.900;100.000;0.250;0.400)	11944.07780	(1.00016;-0.0814;0.20000;0.49998)	0.00018	38
10	(0.900;100.000;0.300;0.650)	49162.30685	(1.00065;-0.3273;0.20001;0.49996)	0.00232	45
11	(1.100;150.000;0.300;0.350)	31723.82659	(0.99915;0.7432;0.20000;0.50015)	0.01067	33
12	(1.100;100.000;0.100;0.300)	766640.94429	(1.00377;-1.8400;0.20007;0.49988)	0.07914	34
13	(0.850;-50.000;0.250;0.550)	50711.01411	(0.99917;0.7026;0.20001;0.50013)	0.00966	43
14	(0.700;-200.00;0.350;0.550)	128355.02291	(1.00000;-0.0553;0.20001;0.49986)	0.00933	48

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