

SPARSE PACKING AND ITS APPLICATIONS¹

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***Abstract:** Sparse packing problem is considered. This problem is motivated by the thermal energy method applied for cleaning complex shaped parts (objects) obtained by additive technologies (3D printing) from particles of non-sintered powder. The sparse packing is aimed to place the objects as distant as possible, freely sliding and rotating on the horizontal shelves of the cylindrical container subject to balancing conditions. A corresponding nonlinear programming model is derived, using the phi-function technique. A solution algorithm is proposed and computational results are presented.*

***Keywords:** sparse packing, geometric objects, balancing conditions, nonlinear optimization, thermal deburring*

Packing problems consist in allocating a set of objects entirely into a given container without overlapping between the objects and the boundary of the container. In optimized packing a certain criterion has to be optimized. Typical examples are minimizing waste or maximizing the number of the packed objects. Packing problems are classified as NP-hard [1]. Further variants of the problem may consider additional constraints, e.g. weight distribution, cargo stability, balance constraints (see. e.g., [2], [3]).

Optimized packing problems have various applications: filling cylindrical nanotubes (pores, capillaries) with nonspherical particles,

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packing powder particles under vibrations, creating void systems for light-weighted parts in 3D printing (see, e.g. , [4], [5]) .

While regular 3D packing focuses on, basically, convex shapes (spheres, convex polyhedra), irregular packing deals with more sophisticated and/or non-convex figures (see, e.g. [6]-[10]).

In most of optimized packing approaches mentioned above, the objective is to find the dense packing. That is, the packing resulting in the smallest unused space of the container or, in the case of an open problem, giving the minimal volume container. However, there are many modern industrial technologies where the concept of the dense packing is not suitable.

Balanced packing 3D objects arises, e.g., in processing parts by the thermal energy method [11]. Thermal energy method is applied for cleaning complex shaped parts obtained by 3D printing from particles of non-sintered powder (see, e.g., [12] , [13]). The material to be removed is burned due to a chemical reaction between the material and the mixture of a combustible gas and the oxygen. For this purpose, the parts are located on a special jig in a cylindrical bell-shaped deburring chamber which is hydraulically closed by a closing plate. The overall workspace is divided into sub-containers by horizontal circular “shelves” rigidly fixed on a thin cylindrical rod passing through the center of the jig. The distance between the shelves depends on the height of the parts to be placed on a specific shelve.

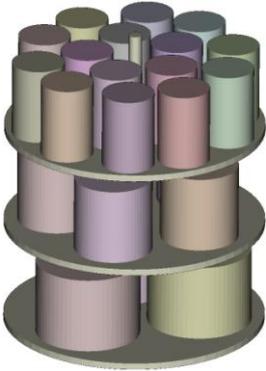
Due to shock waves arising from explosive combustion of the gas mixture in the chamber, the heat fluxes acting on the surfaces of parts can

differ significantly. To ensure uniform distribution of thermal and power effects, sufficiently large distances between the objects, as well as between the object and the container have to be maintained ([14]).

In this research the sparse packing of 3D objects is considered, where the objects are placed in the container as distant as possible. More specifically, the minimal Euclidean distance between the objects as well as between the objects and the boundary of the container is maximized. This new problem statement is motivated by thermal deburring technology, where the objects have to be sufficiently distant to ensure uniform distribution of thermal and power effects. The sparse packing of 3D objects in a vertical cylindrical container divided by the horizontal shelves is considered subject to balancing constraints. The 3D objects may have regular or irregular shapes and can be freely translated and rotated sliding on the shelves. Using the phi-function [15] and quasi-phi-function [16] techniques to state the placement constraints, a mathematical model is formulated and a corresponding nonlinear programming problem is stated. A solution algorithm is proposed and computational results are presented to illustrate the approach.

Figure 1 illustrates examples of local optimal packings of basic and composed 3D objects.

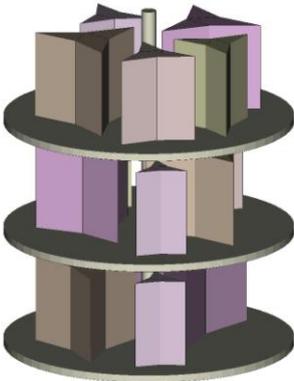
Figure 2 illustrates examples of local optimal packings of the objects that are similar to those presented in, e.g., Custom Design Components [17] and [18].



(a)



(b)

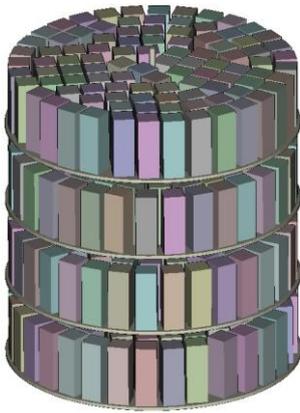


(c)

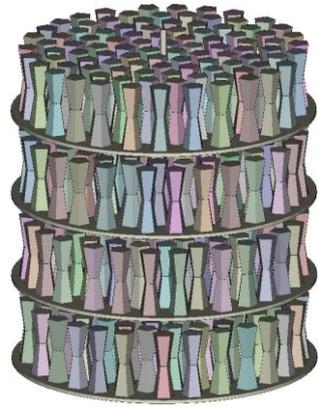


(d)

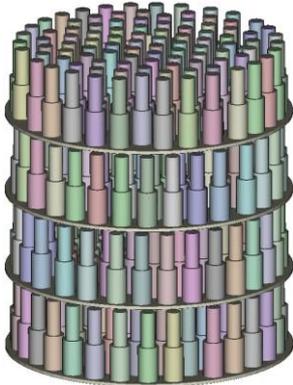
Figure 1. Examples of packing of 3D objects : a) cylinders; c) non-convex prisms; b) convex truncated pyramids;d) non-convex objects



(a)



(b)



(c)

Figure 2. The sparse packing of $N = 400$ objects: a) cuboids, b) non-convex objects composed by the right prisms; c) non-convex objects composed by circular cylinders

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