OPTIMAL CONTROL OF DYNAMIC PROCESSES IN A HEAT EXCHANGER

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Se consideră un model numeric pentru studierea proceselor dinamice într-un schimbător de căldură tub-în-tub. Cu ajutorul soluțiilor numerice obținute se efectuează o abordare pentru stabilirea controlului asupra menținerii temperaturii apei reci la un nivel constant la ieșirea dispozitivului.

Cuvinte cheie: *schimbător de căldură, control optimal.*

Numerical model for study the dynamic processes in a tube-in-tube heat exchanger is developed. Using the obtained numerical solutions we tried to develop an approach for establishing the control over maintaining the temperature of cold water at a constant level at the output of the device.

Keywords: *heat exchanger, optimal control.*

It is considered a tube-in-tube heat exchanger, the principle of operation of which is based on the constant contact of the coolant with the treated liquid. It is used in technological systems for heating or cooling a coolant with a small heat exchange surface in the gas, oil, petrochemical and chemical industries. Heat exchangers with such a design are also used in the food industry, for example, in winemaking and in the dairy production. The final aim of the study is to develop the algorithmic strategy that gives the possibility to maintain the temperature of the cold water at the output of the device at a constant level. To implement such a control, it seems necessary to solve the optimal control problem. Such a problem meets a set of difficulties: the mathematical model is represented by partial differential equations (rather than ordinary ones), the control parameters are in the coefficients at the derivatives (and not in the right-hand side or boundary conditions, as usual), etc.

The mathematical model of the dynamic process of transferring heat energy in devices of this type is presented in many publications [1-3]. The model of dynamical problem includes a system of three differential equations for the temperatures of cold water (heated) $T_l(x,t)$, hot water (heating) $T_h(x,t)$ and the temperature of dividing wall $T_w(t)$

$$\begin{cases} \rho_l c_l D_l \dfrac{\partial T_l}{\partial t} - G_l c_l L \dfrac{\partial T_l}{\partial x} + \alpha_l \Pi_l (T_l - T_w) = 0 \\ m_w c_w \dfrac{\partial T_w}{\partial t} = \alpha_h \Pi_h (T_h - T_w) + \alpha_l \Pi_l (T_l - T_w) \\ \rho_h c_h D_h \dfrac{\partial T_h}{\partial t} + G_h c_h L \dfrac{\partial T_h}{\partial x} - \alpha_h \Pi_h (T_w - T_h) = 0. \end{cases}$$

The constants included in the equations describe the physical and geometric parameters of the given device [1]. This system with specified coefficients and with following boundary and initial conditions

$$T_l(x,0) = T_h(x,0) = T_w(0) = 20°C, \qquad x \in [0, L],$$
$$T_h(0,t) = T_h^0 = 60°C, \quad T_l(L,t) = T_l^L = 30°C, \qquad t \geq 0$$

is solved numerically using the ideas of finite difference method. For this purpose a stable and converging difference scheme is constructed, that gives a possibility to find approximate solutions for discrete times for two equations of the system. In this case, the third equation (for temperature of dividing wall) becomes an ordinary differential equation, the solution of which can be obtained in an analytical form. As it follows from the structure of the initial equations, the model contains dissipative terms. This leads to the fact that the solution to the dynamic problem enters a stationary mode determined by the solution of the static problem. The static problem, being a special case of the original dynamic problem, is a system of two ordinary differential equations and one algebraic equation connecting unknown temperatures. The solution to such a system with given boundary conditions can be obtained in the analytical form.

The purpose of our research is to maintain a constant temperature of cold water at the output $T_l(0,t) = T_l^c$, where T_l^c is a given constant temperature. In the case when the temperature of cold water at the input $T_l(L,t)$ and the mass flow rate of cold water G_l are constants, the problem is simple to solve. Namely, the stationary problem is solved with the input values $T_l(L,t)$ and G_l, and the dynamic problem is solved with the found necessary values $T_h(0,t)$ and G_h. In the case when the parameters of cold water at the input are variable, then in order to maintain a constant temperature of cold water at the output, it is necessary to solve the problem of optimal control, which is formulated as follows. It is required to find a vector control function

$$u(t) = [T_h(0,t), G_h(0,t)] = \left[T_h^0(t), G_h^0(t)\right],$$

which minimizes the value of the functional

$$J(u) = \int_0^T (T_l(0,t) - T_l^c)^2 dt$$

with equations of state

$$\begin{cases} \rho_l c_l D_l \dfrac{\partial T_l}{\partial t} - G_l c_l L \dfrac{\partial T_l}{\partial x} + \alpha_l \Pi_l (T_l - T_w) = 0 \\ m_w c_w \dfrac{\partial T_w}{\partial t} = \alpha_h \Pi_h (T_h - T_w) + \alpha_l \Pi_l (T_l - T_w) \\ \rho_h c_h D_h \dfrac{\partial T_h}{\partial t} + G_h c_h L \dfrac{\partial T_h}{\partial x} - \alpha_h \Pi_h (T_w - T_h) = 0 \\ \dfrac{\partial G_h}{\partial t} + \gamma_h \dfrac{\partial G_h}{\partial x} = 0, \gamma_h = \dfrac{G_h}{\rho_h S_h} \end{cases}$$

and with following boundary and initial conditions

$$T_l(x,0) = T_h(x,0) = T_w(x,0) = T^0, \qquad G_h(x,0) = G_{h0}, x \in [0,L],$$
$$T_h(0,t) = T_h^0(t), G_h(0,t) = G_h^0(t), T_l(L,t) = T_l^L, t \in [0,T].$$

In the last formulated problem, in comparison with the original, one more equation is added for $G_h(x,t)$, since during the control process we can change the value of the mass flow rate of hot water only at the input point, i.e. at $x = 0$.

A series of numerical experiments was carried out in order to refine the mathematical model and identify the main control variables and control law.

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