ROBUST CONSTRUCTIONS OF RISK MEASURES FOR OPTIMIZATION UNDER UNCERTAINTY

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Summary. *For the case when the measure of the initial probability space is unknown and is described by an uncertainty set, robust constructions of risk measures are proposed. Their calculation for polyhedral coherent risk measures, as well as portfolio optimization with their participation, are reduced to appropriate linear programming problems.*
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In [1], the concept of a coherent risk measure (CRM) $\rho(.)$ for random variables (r.v.) of a probability space (Ω, Σ, P) was introduces and its dual representation was proved in the form

$$\rho(X) = \sup\left\{\int_\Omega \zeta(\omega) X(\omega) dP(\omega) : \zeta \in M\right\}, \quad (1)$$

where M is some convex and weakly* closed set of probability densities, i.e.

$$\zeta \in M_0 = \left\{\zeta(.) \geq 0 \; a.s., \int_\Omega \zeta(\omega) dP(\omega) = 1\right\}. \quad (2)$$

Consider the following definition of polyhedral CRM (PCRM), which allows us to extend this concept, introduced in [2] for discretely distributed r.v., to the general case.

Definition 1. A risk measure of the form (1) is called PCRM if set M is either set (2), or its intersection with any of the following two sets:

$$M_1 = \left\{\zeta : \int_\Omega \zeta(\omega) X_i(\omega) dP(\omega) \leq c_i, i = 1, ..., k\right\} \quad (3)$$

for some r.v. X_i and real values c_i, and

$$M_2 = \{\zeta : \zeta(.) \leq \gamma \; a.s.\} \quad (4)$$

For some $\gamma > 0$. In this case, M_0 is called the standard part of the description of set M.

Consider examples of similar PCRM:

1) maximum losses: $\rho(X) = ess \sup X$, then $M = M_0$;

2) average losses:

$\rho(X) = E_P[X]$, in this case $M = M_0 \cap M_2 = \{\zeta(.) = 1 \ a.s.\}$;

3) Conditional Value-at-Risk [3]: $\rho(X) = CVaR_\alpha(X)$, then

$$M = M_0 \cap M_2 = \left\{0 \le \zeta(.) \le 1/(1-\alpha) \ a.s., \int_\Omega \zeta(\omega) dP(\omega) = 1\right\}. \quad (5)$$

Note that set M_1 from (3) can be interpreted as a system of inequalities for the first moments of r.v. $X_i, i = 1,...,k$.

Definition 2. If the initial probability measure P is not known and is described by an uncertainty set U in the form $P_0 \in U$, then the robust construction of measure $\rho(.)$ from (1) is called

$$\rho_U(X) = \sup_{P_0 \in U} \rho(X) = \sup_{P_0 \in U} \sup \left\{\int_\Omega \zeta(\omega) X(\omega) dP_0(\omega) : \zeta \in M\right\}. \quad (6)$$

It can be shown that for the case of discrete distributions and a polyhedral uncertainty set U, the calculation of robust PCRM constructions of form (6) is reduced to solving appropriate linear programming problems (LPP).

It is not difficult to pose problems of portfolio optimization in terms of the reward-risk ratio, as well as on maximizing the Sharpe ratio, in which the risk is described by robust PCRM constructions, and the reward function is described by robust analogues of profitability. It can be shown that for discrete distributions and polyhedral set U, both classes of such portfolio problems are reduced to appropriate LPPs, which greatly facilitates the search for their solutions.

Literature.

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