THE STOCHASTIC PROBLEM FOR CLOUD SERVICES

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Abstract. *Optimization can be applied in developing profitability management tools for a cloud service broker working according to a certain business model. On behalf of the managing telecommunications holding company (telecommunications operator), this broker integrates, aggregates and configures software and data storage services of third-party Internet software vendors. Such a broker receives only fixed commissions from this company, based on the subscription fee, but does not pay royalties to an Internet software vendor and does not receive payments from the sale of service packages.*
Key words: *cloud broker, service bundle, random demand*.

The development of computer architectures has been motivated and is motivated by practical applications [1], which today have reached the level of creating new valuable virtual and tangible assets [2]. Distributed information technology (розподілена інформаційна технологія, РІТ; RIT) of scientific and organizational activity (науково-організаційної діяльності, НОД; NOD) of the National Academy of Sciences of Ukraine [3] at the present stage is developing as a cloud architecture, also able to generate new assets, for example, objects of intellectual property. At the same time, RIT NOD should meet modern challenges to cloud architectures.

The cloud broker faces the problem of limited human resources required to carry out the relevant legal, technical and economic activities [4]. In addition, the broker faces the problem of uncertainty in sales, service prices, the share of resource use, or the risk of losing operational and financial goals [5].

To run a broker's business efficiently, one needs to find services and their bundles that increase profitability and reduce financial risk by solving certain optimization problems. Information on such services is needed to support negotiations on fixed and variable commissions, as well as to prioritize services and their packages to be provided [6]. Thus, for the cloud services broker, both profitability management tools and services portfolio development tools are useful. In general, a cloud service broker is an organization that negotiates the relationships between cloud service clients and Internet software vendors [7]. Cloud broker can be created on the basis of different business models regarding the type of service (platform, infrastructure, software), type of clients (enterprise, household), functions performed (identity management, accounting, billing, location, etc.), the degree of rebranding, measures of aggregation of services and other criteria [8].

We introduce a binary variable x_O whose value is equal to 1 (if the cloud broker offers services to the seller of services − a client of the market or business unit that generates income) or 0 (if the cloud broker does not offer services).

Let the broker have N_C clients who sell service packages created by the cloud broker. The binary variable $x_{C,g}$ indicates whether the cloud broker will service the svendor $g = 1,\dots,N_C$. The constraints

$$x_{C,g} \le x_O,\ g = 1,\dots,N_C, \qquad (1)$$

guarantee the inactivity of service sellers in the inactivity of the broker. Assume, only one service vendor h can sell in the local market:

$$x_{M,h} \le x_{C,g},\ g = 1,\dots,N_C, \qquad (2)$$

for all h in the set $X_{MC,g}$ of market ISVs that could potentially be included in the service portfolio in a market g, where the variable $x_{M,h}$ is binary. Market ISVs are not available if they are not technologically integrated on the platform:

$$x_{M,h} \le x_{V,k},\ k = 1,\dots,N_V, \qquad (3)$$

for all h in the set $X_{MV,k}$ of market ISVs that can be generated from a global ISV k, where the variable $x_{V,k}$ is binary. Market-oriented services are not available if the corresponding local market ISV is not implemented on the platform:

$$x_{S,i} \le x_{M,h}, \; h = 1,\ldots,N_M, \quad (4)$$

for all i in the set $X_{SM,h}$ of services provided by the local market ISV h. Because individual services belong to groups of services that require common costs for integration on the platform, individual services are not available if the relevant groups are not integrated on the platform:

$$x_{S,i} \le x_{G,j}, \; j = 1,\ldots,N_G, \quad (5)$$

for all i in the set $X_{SG,j}$ of services in the group j. A service bundle cannot exist if ancillary services are not installed on the platform:

$$x_{B,l} \le x_{S,i}, \; i = 1,\ldots,N_S, \quad (6)$$

for all l in the set $X_{BS,i}$ of potential service packages that the service i can promote.

The total income (profit) of the cloud broker is given by function

$$\Pi = \pi_O x_O + \sum_{g=1}^{N_C} \pi_{C,g} x_{C,g} + \sum_{i=1}^{N_S} \pi_{S,i} x_{S,i} + \sum_{l=1}^{N_B} \pi_{B,l} x_{B,l} + \sum_{h=1}^{N_M} \pi_{M,h} x_{M,h} + \sum_{j=1}^{N_G} \pi_{G,j} x_{G,j}, \quad (7)$$

where Π, π_O, $\pi_{C,g}$, $\pi_{S,i}$, $\pi_{B,l}$, $\pi_{M,h}$, $\pi_{G,j}$ – net income of the relevant entity (participant), which is computed on the basis of income and expenses (current values based on cash inflows and outflows).

Denote Q_T the available quantity of time and denote q_O, $q_{C,g}$, $q_{S,i}$, $q_{B,l}$, $q_{M,h}$, $q_{G,j}$ the amount of time use by the relevant subject:

$$Q_T \ge q_O x_O + \sum_{g=1}^{N_C} q_{C,g} x_{C,g} + \sum_{i=1}^{N_S} q_{S,i} x_{S,i} + \sum_{l=1}^{N_B} q_{B,l} x_{B,l} + \sum_{h=1}^{N_M} q_{M,h} x_{M,h} + \sum_{j=1}^{N_G} q_{G,j} x_{G,j}. \quad (8)$$

Thus, the optimization model of the cloud broker can be formulated as the maximization of the objective function (7) for the binary variables x_O, $x_{C,g}$, $x_{S,i}$, $x_{B,l}$, $x_{M,h}$, $x_{G,j}$ under the constraints (1)–(6), (8). For brevity of the problem of maximizing the function (7) with restrictions (1)−(6), (8) we introduce vector notation:

$$\vec{\pi}_C = (\pi_{C,1},\dots,\pi_{C,N_C}), \quad \vec{\pi}_V = (\pi_{V,1},\dots,\pi_{V,N_V}), \quad \vec{\pi}_M = (\pi_{M,1},\dots,\pi_{M,N_M}),$$

$$\vec{\pi}_G = (\pi_{G,1},\dots,\pi_{G,N_G}), \quad \vec{\pi}_S = (\pi_{S,1},\dots,\pi_{S,N_S}), \quad \vec{\pi}_B = (\pi_{B,1},\dots,\pi_{B,N_B}),$$

$$\vec{q}_C = (q_{C,1},\dots,q_{C,N_C}), \quad \vec{q}_V = (q_{V,1},\dots,q_{V,N_V}), \quad \vec{q}_M = (q_{M,1},\dots,q_{M,N_M}),$$

$$\vec{q}_G = (q_{G,1},\dots,q_{G,N_G}), \quad \vec{q}_S = (q_{S,1},\dots,q_{S,N_S}), \quad \vec{q}_B = (q_{B,1},\dots,q_{B,N_B}),$$

$$\vec{x}_C = (x_{C,1},\dots,x_{C,N_C}), \quad \vec{x}_V = (x_{V,1},\dots,x_{V,N_V}), \quad \vec{x}_M = (x_{M,1},\dots,x_{M,N_M}),$$

$$\vec{x}_G = (x_{G,1},\dots,x_{G,N_G}), \quad \vec{x}_S = (x_{S,1},\dots,x_{S,N_S}), \quad \vec{x}_B = (x_{B,1},\dots,x_{B,N_B}),$$

$$\vec{\pi} = (\pi_O, \vec{\pi}_C, .\vec{\pi}_V, \vec{\pi}_M, \vec{\pi}_G, \vec{\pi}_S, \pi_B),$$

$$\vec{q} = (q_O, \vec{q}_C, .\vec{q}_V, \vec{q}_M, \vec{q}_G, \vec{q}_S, \vec{q}_B),$$

$$\vec{x} = (x_O, \vec{x}_C, .\vec{x}_V, \vec{x}_M, \vec{x}_G, \vec{x}_S, \vec{x}_B).$$

Then constraint (8) can be written as

$$\vec{q}\,\vec{x}^T \le Q_T, \tag{9}$$

where $\vec{x}^T$ is transposed to the vector $\vec{x}$. Restrictions (1)−(6) can be written as

$$A\vec{x}^T \le \vec{0}, \tag{10}$$

where A is some (sparse) matrix, all elements of each row of which are equal to 0, except for an element equal to 1 and an element equal to -1. Therefore, the maximization problem of function (7) under constraints (1)−(6), (8) can be rewritten as a maximization problem

$$\Pi = \vec{\pi}\,\vec{x}^T \tag{11}$$

by the binary $\vec{x}$ vector under inequalities (9), (10). Let be ω a random event from a set Ω of possible future states, which occurs with some probability α_ω. Then the expected current value is maximized

$$E_\omega[\Pi_\omega] = E[\vec{\pi}_\omega]\vec{x}, \tag{12}$$

instead of the objective function (11), and it is used

$$E_\omega[\vec{q}_\omega]\vec{x}^T \le Q_T. \tag{13}$$

instead of constraint (9) [9]. If φ is the cost (penalty) per unit of non-compliance with the restriction (13), then instead of the current cost (12) and the restriction (13) one can enter the criterion

$$E_\omega[\Pi_\omega] = E_\omega[\vec{\pi}_\omega]\vec{x} - E_\omega\left[\varphi \max\{\vec{q}_\omega\vec{x}^T - Q_T; 0\}\right]. \tag{14}$$

In addition, one can enter a parametric limit on the excess (exceedance) of resources:

$$\max\{\vec{q}_\omega \vec{x}^T - Q_T ; 0\} \le R_T . \qquad (15)$$

For criterion (14) and inequality (10) for a binary vector instead of constraint (15) one can enter a constraint

$$E_\omega \left[\max\{Q_F - \vec{\pi}_\omega \vec{x} - \varphi \max\{\vec{q}_\omega \vec{x}^T - Q_T ; 0\}; 0\}\right] \le R_F , \qquad (16)$$

where Q_F is the target value of the current value, R_F is the parameter whose value is selected by the user. If each scenario $s = 1,...,S$ has a probability α_s, then the maximization problem of criterion (14) under constraints (10), (16) can be rewritten as a maximization problem

$$E_s[\Pi_s] = \sum_{s=1}^{S} \alpha_s (\vec{\pi}_s \vec{x} - v_s \varphi) \qquad (17)$$

by binary vector $\vec{x}$, non-negative vectors $\vec{v} = (v_1,...,v_S) \ge \vec{0}$, $\vec{w} = (w_1,...,w_S) \ge \vec{0}$, with restrictions (10),

$$\vec{q}_s \vec{x}^T - v_s \le Q_T , \ s = 1,...,S , \qquad (18)$$

$$\vec{\pi}_s \vec{x} - v_s \varphi + w_s \ge Q_F , \ s = 1,...,S , \qquad (19)$$

$$\sum_{s=1}^{S} \alpha_s w_s \le R_F . \qquad (20)$$

Since in practice it can be difficult for the user to set the value of the parameter R_F, especially when changing the input data or the number of subjects, it is more convenient to use a convex combination instead of the objective function (17) and constraint (20):

$$(1-\lambda) \sum_{s=1}^{S} \alpha_s (\vec{\pi}_s \vec{x} - v_s \varphi) - \lambda \sum_{s=1}^{S} \alpha_s w_s , \qquad (21)$$

where $\lambda \in (0, 1)$. Under fairly general assumptions, any solution of problem (10), (17)−(20) is also a solution of problem (10), (18), (19), (21) for some R_F. In turn, the convex combination (21) can be modified using CVaR:

$$(1-\lambda) \sum_{s=1}^{S} \alpha_s (\vec{\pi}_s \vec{x} - v_s \varphi) - \frac{\lambda}{\alpha_{VaR}} \sum_{s=1}^{S} \alpha_s w_s - \lambda V .$$

Different cloud brokers have different attitudes to choice of important solutions for their businesses. Solutions can relate to pricing, capacity planning and utilization in combination with service quality, security, scalability and other issues.

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