OPTIMAL INVENTORY MANAGEMENT
IN CONDITION OF UNCERTAINTY

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Abstract. Two inventory management strategies, – maximization of demand fulfillmet and maximization of profitability, – are compared.
Key words: inventory management, marginal analysis, effective strategy for small competitor.

Given uncertain market demand case, maintaining the optimal level of inventory at which both, – the surplus of purchased products and the shortage – are minimized is one of the main tasks of the inventory management. Effective solutions maximize company's profitability by suggesting optimum stock level, under which both, – the financial resources "frozen" in surplus and the lost sales are minimized [1]. Advance in technologies of Machine Learning (ML) [2] lead to new reality when inventory managers of large (and financially well established) retailers could require higher and higher rate of demand fulfillment. Small competitors on other side could find lucrative market niche for them in trendy goods segment by balancing cost of shortage and cost of excess. Thses two strategies are compared in following R script implementation (table1).

Suppose expected demand (based on different users' preliminary interest assessment) is estimated at 29972 units and has standard deviation of 12135 units. To estimate expected demand "empavg" package, developed within project "Analytical methods and machine learning in control theory and decision-making in conditions of conflict and uncertainty" funded by National Research Foundation of Ukraine was used [3].

Inventory managers under strategy of high demand fulfillment rate are required to provide sufficient enough stock level. Could assume that such target level is at least 90%. Given characteristics of our demand, estimated target stock level to order from suppliers is 45523 units.

Table 1. R script to compare different inventory management strategies

```
library("empavg")

# parameters
sku_cost <- 11
sku_price <- 24
preliminary_interest <- c(21453, 17392, 22879, 33210, 34099,  9658, 21876,
                          25391, 22275, 33091, 42072, 48605,  6338, 38242,
                          27432, 54979, 15329, 34213, 28467, 51076, 42781,
                          19758, 27855, 26038, 28837, 38764, 17992, 42528)

sku_estimated_demand <- empavg::get_empavg(preliminary_interest,

                                           method = "max_likelihood")[['mu']]
# 29971.39

var_of_estimated_demand <- sd(preliminary_interest) # 12134.72

# Strategy I: satisfying at least 90% order rate (90% of possible demand cases)
# calculating number of skus to order from suppliers
or90_num_of_sku_to_order <- ceiling( # taking next largest integer
  qnorm(0.90, sku_estimated_demand, var_of_estimated_demand)) # 45523

# Strategy II: Profit maximizing
# cost of shortage in case actual demand > ordered number of skus from supplier
c_s <- sku_price - sku_cost # 13

# cost of excess in case actual demand < ordered number of skus from supplier
c_e <- sku_cost # 11

# critical ratio
cr <- c_s / (c_s + c_e) # 0.5416667

# searching optimal number of skus to order from suppliers
# given P[expected_demand <= sku_ordered_num] = cr
opt_num_of_sku_to_order <- ceiling( # taking next largest integer
  qnorm(cr,
        sku_estimated_demand,
        var_of_estimated_demand))
# 31242 # profit maximizing quantity

calc_profit <- function(ordered_num_of_skus, sku_estimated_demand,
                        sku_price, sku_cost){
  sku_profit <- sku_price - sku_cost
  est_profit <- ifelse(ordered_num_of_skus > sku_estimated_demand,
         (sku_estimated_demand * sku_price) - (ordered_num_of_skus * sku_cost),
         (ordered_num_of_skus * sku_profit) -
           ((sku_estimated_demand - ordered_num_of_skus) * sku_profit)
         )
  return(est_profit)
}

# expected profit under optimal quantity
calc_profit(opt_num_of_sku_to_order, sku_estimated_demand,
            sku_price, sku_cost) # 375651.3

# expected profit under strategy of satisfying at least 90% order rate
calc_profit(or90_num_of_sku_to_order, sku_estimated_demand,
            sku_price, sku_cost) # 218560.3
```

Profitability maximizing strategy is based on results of marginal analysis [4, 5]. That is, finding optimal number of SKUs (Q) (stock keeping units) under which excess cost c_e and shortage cost c_s are minimized given probable demand (x) (equations 1-4).

$$c_e P[x \leq Q] = c_s (1 - P[x \leq Q]) \tag{1}$$

$$c_e P[x \leq Q] = c_s - c_s P[x \leq Q] \tag{2}$$

$$c_e P[x \leq Q] + c_s P[x \leq Q] = c_s \tag{3}$$

$$P[x \leq Q] = \frac{c_s}{c_e + c_s} \tag{4}$$

If $E[c_e] < E[c_s]$ should increase Q, and vice versa. Optimal stock level Q^* is reached when $E[c_e] = E[c_s]$. Right hand side of equation 4 is called critical ratio. Inferred from characteristics of our demand and calculated critical ratio (0.54), estimated optimal stock level to order from suppliers under profit maximizing strategy is 31242 units.

Profit here is not just in accountant terms but in economic ones, that is includes also lost sales; and is calculated according to equations 5-8.

$$Profit(Q,x) = \begin{cases} px - cQ \ if \ x \leq Q \\ pQ - cQ - (p-c)(x-Q) \ if \ x > Q \end{cases} \tag{5}$$

$$E[Profit(Q)] = \int_0^\infty P(Q,x) f_x(x) dx \tag{6}$$

$$E[Profit(Q)] = \int_0^Q (px - cQ) f_x(x) dx - \int_Q^\infty (p-c)(x-Q) f_x(x) dx \tag{7}$$

$$E[Profit(Q)] = p \int_0^Q (x) f_x(x) dx - cQ - (p-c) \int_Q^\infty (x-Q) f_x(x) dx \tag{8}$$

Estimated expected profit given optimal stock level of 31242 units is 375651.3. And, expected profit given stock level of 45523 units (with demand fulfillment rate 90%) is 218560.3. That is, only approximately 58% from possible optimal level.

Large companies are trying to keep high demand fulfillment rate because they need to support image of "one stop shop". Also, they could

have special discounts from suppliers given significant size of SKU order volumes.

Under profit maximizing strategy, with optimal 31242 SKUs at stock, with expected average demand of 29972 units and its volatility of 12135 units chances to run out of stock is rather high. However, ultimately, optimal stock level inferred based on shortage and excess costs trade off is great profitable alternative to apply by small agile company which does not need quite high demand fulfillment rate or just perform their trade business via interenet and do not have to keep wide representative shelves of assortment.

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