RESEARCH AND SOLUTION METHODS OF PROBLEMS OF NETWORK STRUCTURE OPTIMIZATION

K.R. AIDA-ZADE,
Institute of Control Systems of Azerbaijan NAS,
kamil_aydazade@rambler.ru

In this paper we study a class of finite-dimensional optimization problems of the Jacobians of functions describing constraints of the type of equalities and inequalities, which are weakly and arbitrarily filled matrices of large dimension.
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Let there be given some object of complex structure M consisting of subobjects M_i. The state of each of the subobjects is determined by the vector $x^i \in R^{n_i}$, $i \in I$. The subobjects M_i are controlled by an object V consisting of subobjects V_j with control actions $w^j \in R^{r_j}$, $j \in J$.

The interconnection between all subobjects M_i and V_j, $i \in I$, $j \in J$, is arbitrary and generally nonlinear:

$$x^i = F\left(X^i, W^i\right),\ i \in I. \tag{1}$$

Here $X^i = \left\{x^s : s \in I_i^+ \subset I\right\}, W^i = \left\{w^s : s \in J_i^+ \subset J\right\}$, I_i^+, J_i^+, $i \in I$ are given index sets that determine the structure (interconnection) of the subobjects M and V. Usually this indicates a weak interconnection between the subobjects.

The problem is to determine the values of actions v^j, $j \in J$ that minimize a given continuously differentiable objective function

$$\Phi_0\left(X,W\right) \to \min, \tag{2}$$

$$X = \left\{x^i : i \in I\right\},\ W = \left\{w^j : j \in J\right\}.$$

There are state constraints on the subobjects M_i and V_j:

$$G^i\left(x^i\right)\le 0,\ i\in I,\ \ Q^j\left(w^j\right)\le 0,\ j\in J\ . \qquad (3)$$

Here $G^i\left(\cdot\right), Q^j\left(\cdot\right), i\in I,\ j\in J$ are given continuously differentiable vector-functions, the dimensions of which are determined by the number of constraints.

The structure of the optimization problem (1) - (3) will be called network. Such problems arise when using decomposition methods employed at the stage of constructing mathematical models of complex objects under study.

Examples of such objects are abundant: artificial neural networks used in recognition systems, pipeline transport networks, mechanisms of complex manipulators and robots, etc.

To solve problems of network structure optimization, it is proposed to use a first-order optimization method, for example, conditional gradient projection methods:

$$W^{k+1}=P_{(3)}\left(W^k-grad_W\Phi_0\left(X^k,W^k\right)\right), k=0,1,...$$

Here $P_{(3)}\left(\cdot\right)$ is the operator of projection onto an admissible set determined by constraints (3).

The formulas for the gradient of the objective function are obtained:

$$\frac{d\Phi_0\left(X,W\right)}{dw^j}=\frac{\partial\Phi_0\left(X,W\right)}{\partial w^j}+\sum_{i\in J_j^-}\frac{\partial F_i\left(X^i,W^i\right)}{\partial w^j}\Psi^i,\ j\in J, \qquad (4)$$

$$\Psi^i=\frac{\partial\Phi_0\left(X,W\right)}{\partial x^i}+\sum_{s\in I_i^-}\frac{\partial F_s\left(X^s,W^s\right)}{\partial x^i}\Psi^s,\ i\in I\ . \qquad (5)$$

Here $J_j^-=\left\{s: j\in J_s^+\right\}, j\in J$, $I_i^-=\left\{s: i\in I_s^+\right\}, i\in I$ are conjugate sets with respect to the corresponding sets J_s^+, I_s^+, Ψ^s - vectors of conjugate variables with respect to the vectors x^i.

Note that the formulas (4),(5) are a generalization of the so-called "backpropagation" method.

The report will provide practical aspects of the application of the proposed approach, related to the solution of various applied problems.