Optimal Packing Circular Cylinders into a Cylindrical Container Taking into Account Behavior Constraints

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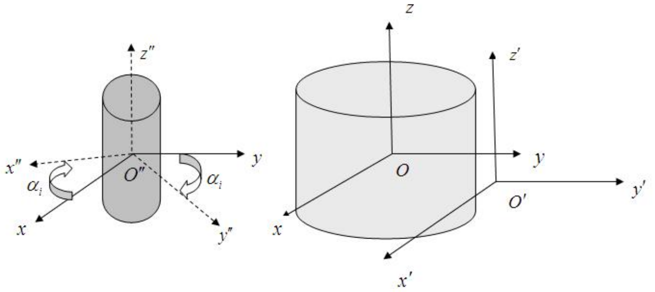
Problem formulation I

Let $\{C_i, i = 1, 2, ..., N\}$ be a set of circular cylinders. Each cylinder C_i given by its radius r_i, height h_i and mass m_i.
Let Ω be a container of cylindrical form of radius R and height $2H$.



We denote container Ω with the set of cylinders inside the container by Ω^A.

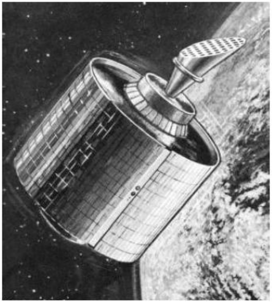
Problem formulation II

Problem.

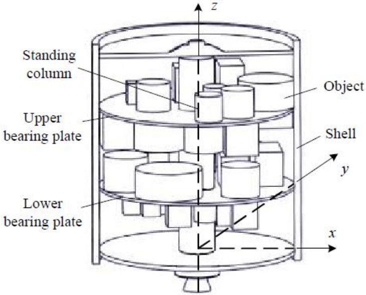
Pack the set of cylinders $\{C_i, i = 1, 2, ..., N\}$ into a container Ω of minimal radius taking into account mechanical behavior constraints (balance, inertia moments, stability) of system Ω^A.

Application

These problems have a wide spectrum of applications in space engineering for satellite modeling [Fasano,Pinter(2012)].



a. the international communication satellite



b. the simplified satellite module

FASANO G., PINTER J.D., EDS., Modeling and Optimization in Space Engineering. Springer Optimization and Its Applications. – Springer. – New York. – 2012. – 404 p.

Application

The packing problem is considered in the paper

Chao Che, Yi-shou Wang, Hong-fei Teng. Test problems for quasi-satellite packing: Cylinders packing with behavior constraints and all the optimal solutions known. (Online), (2008)

In order to solve the problem authors use a heuristic algorithm.

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Mathematical model I

We present a mathematical model of the problem as a constraint optimization problem using phi-function technique

$$F(u^*) = \min_{u \in W \subset R^{3N+1}} F(u), \tag{1}$$

$$W = \left\{ \begin{array}{c} u \in R^{3N+1} : \Phi_k(u) \geq 0, k = 1, \ldots, N(N+1)/2, \\ G_1(u) \geq 0, G_2(u) \geq 0, G_3(u) \geq 0, \\ R \geq r_i, i = 1, \ldots, N, \end{array} \right\} \tag{2}$$

where
$F(u) = R$ is an objective function,
$u = (u_1, u_2, ..., u_N, R)$ is a vector of variables,
$u_i = (x_i, y_i, z_i)$ is a vector of placement parameters of cylinder,
W is a feasible region

Mathematical model II

The feasible region W is formed by **placement** constraints and **behavior** constraints.

Placement constraints use phi-functions for description of non-overlapping and containment constraints

$$\Phi_{ij}^{CC} \geq 0, i > j = 1, ..., N-1, \quad \Phi_i^{\Omega^* C} \geq 0, i = 1, ..., N$$

where Φ_{ij}^{CC} is phi-function for two cylinders C_i and C_j, $\Phi_i^{\Omega^* C}$ is phi-function for cylinder C_i and object $\Omega^* = E^3 \backslash int\Omega$.

Mathematical model III

Behavior constraints include:

- balance constraints $G_1(u) \geq 0$,
- inertia moment constraints $G_2(u) \geq 0$,
- stability constraints $G_3(u) \geq 0$.

Let us consider behavior constraints in details

Balance constraints I

Center of mass (x_c, y_c, z_c) of the set $\{C_i, i \in I_N\}$ is defined as:

$$x_c = \frac{\sum_{i=1}^{N} m_i x_i}{\sum_{i=1}^{N} m_i}, \quad y_c = \frac{\sum_{i=1}^{N} m_i y_i}{\sum_{i=1}^{N} m_i}, \quad z_c = \frac{\sum_{i=1}^{N} m_i z_i}{\sum_{i=1}^{N} m_i}.$$

(x_e, y_e, z_e) is a center of mass of Ω^A, which coincide with the symmetry centre of container Ω. We assume that $(x_e, y_e, z_e) = (0, 0, 0)$.

Balance constraints II

Balance constraints have the following form:

$$G_1(u) \geq 0,$$

where

$$G_1(u) = \min\{g_1(u), g_2(u), g_3(u)\},$$

$$g_1(u) = \min\{-(x_e - x_c) + \Delta x_c, (x_e - x_c) + \Delta x_c\},$$

$$g_2(u) = \min\{-(y_e - y_c) + \Delta y_c, (y_e - y_c) + \Delta y_c\},$$

$$g_3(u) = \min\{-(z_e - z_c) + \Delta z_c, (z_e - z_c) + \Delta z_c\},$$

$(\Delta x_c, \Delta y_c, \Delta z_c)$ – allowable deviations from point (x_e, y_e, z_e) of center of mass of Ω^A

Inertia moment constraints I

Inertia moments $J_x(u), J_y(u), J_z(u)$ of system Ω^A are defined as

$$J_x(u) = \sum_{i=1}^{N} J''_{xi} + \sum_{i=1}^{N} m_i(y_i^2 + z_i^2) - (y_c^2 + z_c^2) \sum_{i=1}^{N} m_i,$$

$$J_y(u) = \sum_{i=1}^{N} J''_{yi} + \sum_{i=1}^{N} m_i(x_i^2 + z_i^2) - (x_c^2 + z_c^2) \sum_{i=1}^{N} m_i,$$

$$J_z(u) = \sum_{i=1}^{N} J''_{zi} + \sum_{i=1}^{N} m_i(x_i^2 + y_i^2) - (x_c^2 + y_c^2) \sum_{i=1}^{N} m_i.$$

where

$$J''_{xi} = J''_{yi} = \frac{1}{12} m_i(3r_i^2 + 4h_i^2), J''_{zi} = \frac{1}{2} m_i r_i^2$$

Inertia moment constraints II

Inertia moment constraints have the following form:

$$G_2(u) \geq 0,$$

where

$$G_2(u) = \min\{g_4(u), g_5(u), g_6(u)\},$$

$$g_4(u) = \min\{-J_x(u) + \Delta J_x, J_x(u) + \Delta J_x\},$$

$$g_5(u) = \min\{-J_y(u) + \Delta J_y, J_y(u) + \Delta J_y\},$$

$$g_6(u) = \min\{-J_z(u) + \Delta J_z, J_z(u) + \Delta J_z\},$$

$(\Delta J_x, \Delta J_y, \Delta J_z)$ – allowable deviations from inertia moment of Ω^A

Stability constraints I

Angle deviations $\varphi_x(u), \varphi_y(u), \varphi_z(u)$ of the main inertia axis of the system from axis of the fixed coordinate system are defined by the following relations:

$$\varphi_x(u) = \frac{1}{2} arctg\left(\frac{2J_{xy}(u)}{J_y(u) - J_x(u)}\right), \quad J_{xy}(u) = \sum_{i=1}^{N} m_i x_i y_i - x_c y_c \sum_{i=1}^{N} m_i,$$

$$\varphi_y(u) = \frac{1}{2} arctg\left(\frac{2J_{yz}(u)}{J_z(u) - J_y(u)}\right), \quad J_{yz}(u) = \sum_{i=1}^{N} m_i y_i z_i - y_c z_c \sum_{i=1}^{N} m_i,$$

$$\varphi_z(u) = \frac{1}{2} arctg\left(\frac{2J_{xz}(u)}{J_z(u) - J_x(u)}\right), \quad J_{xz}(u) = \sum_{i=1}^{N} m_i x_i y_i - x_c y_c \sum_{i=1}^{N} m_i.$$

Stability constraints II

Stability constraints have the following form:

$$G_3(u) \geq 0,$$

where

$$G_3(u) = \min\{g_7(u), g_8(u), g_9(u)\},$$

$$g_7(u) = \min\{-\varphi_x(u) + \Delta\varphi_x, \varphi_x(u) + \Delta\varphi_x\},$$

$$g_8(u) = \min(-\varphi_y(u) + \Delta\varphi_y, \varphi_y(u) + \Delta\varphi_y\},$$

$$g_9(u) = \min\{-\varphi_z(u) + \Delta\varphi_z, \varphi_z(u) + \Delta\varphi_z\},$$

$\Delta\varphi_x, \Delta\varphi_y, \Delta\varphi_z$ are allowable errors of angles
$\varphi_x(u), \varphi_y(u), \varphi_z(u)$

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Solution method I

Step 1. We generate a new function

$$f(u) \;=\; R + P_1 \sum_{k=1}^{n} \max\{0, -\Phi_k\} + P_2 \sum_{k=n+1}^{n+18} \max\{0, -g_k\} + P_3 \max\{0, -R + \max_{i=1,\dots,N} r_i\},$$

by means of non-smooth penalty P_1, P_2, P_3, $n = N(N+1)/2$, Φ_k are given phi-functions and g_k are given behavior functions

Step 2. We reduce problem (1)–(2) to the following non-constrained nonsmooth optimization problem

$$\min_{u \in E^{3N+1}} f(u). \tag{3}$$

Solution method II

In order to realize the model (3) we

1) generate a set of random starting points
2) apply Shor's r-algorithm[1] for each starting point to search for a local minima

1

Shor N.Z. Nondifferentiable optimization and polynomial problems. – Boston; Dordrecht; London: Kluwer Academic Publishers, 1998. – 412 p.

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Simple test problem

We show our results with help of the simple test problem:

- $N = 5$
- $H = 1,\ h_i = 1,\ i = 1, ..., 5$

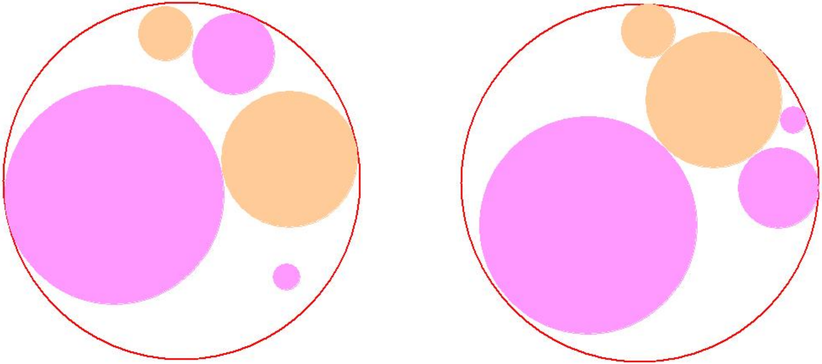
Cylinders have different radii and masses:

- $r_1 = 0.1,\ r_2 = 0.2,\ r_3 = 0.3,\ r_4 = 0.5,\ r_5 = 0.8$
- $m_1 = 0.0785,\ m_2 = 0.314,\ m_3 = 0.7065,\ m_4 = 1.9625,$
  $m_5 = 5.024$

System behavior parameters:

- $(x_e, y_e, z_e) = (0, 0, 0)$
- $(\Delta x_c, \Delta y_c, \Delta z_c) = (0.0001, 0.0001, 0.0001)$
- $(\Delta J_x, \Delta J_y, \Delta J_z) = (5, 5, 5)$

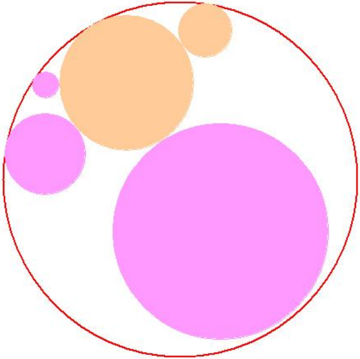
Optimal placement of cylinders (view from above)



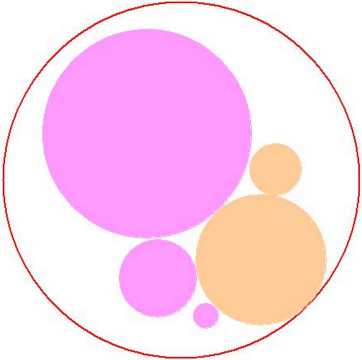
$F(u^*) = R^* = \mathbf{1.3000}$ is the global minimum of the problem without behavior constraints

$F(u_1^*) = R^* = \mathbf{1.3161}$ is the global minimum of the problem with balance constraints

Optimal placement of cylinders (view from above)



$F(u_2^*) = R^* = \mathbf{1.3161}$ is the global minimum of the problem with balance constraints



$F(u^*) = R^* = \mathbf{1.3625}$ is the global minimum of the problem with balance and inertia moment constraints.

Our perspectives

to test our algorithm for medium and large size problems

Thanks

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Question?

Thank you for your attention

BACK UP SLIDES: Phi-functions

phi-functions for 3D-case

$$\Phi_{ij}^{CC} = \max\{(x_j - x_i)^2 + (y_j - y_i)^2 - (r_i + r_j)^2, \\ z - (h_i + h_j), -z - (h_i + h_j)\}, \quad (1)$$

$$\Phi_i^{\Omega^* C} = \min\{-x_i^2 - y_i^2 + (R - r_i)^2, -z + (H - h_i), \\ z + (H - h_i)\} \quad (2)$$

phi-functions for 2D-case ($H = h_i$)

$$\Phi_{ij}^{CC} = (x_j - x_i)^2 + (y_j - y_i)^2 - (r_i + r_j)^2, \quad (3)$$

$$\Phi_i^{C^* C} = -x_i^2 - y_i^2 + (R - r_i)^2, \quad (4)$$