POLYNOMIAL OPTIMIZATION
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T. Bardadym

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OUTLINE

- Event
- Main participants

The 2013 Newton Institute Programme on polynomial optimization

This special issue of Mathematical Programming Series B was originally conceived during a 4-week residential programme on PO which took place in July and August 2013 at the Isaac Newton Institute for the Mathematical Sciences, an internationally recognised research institute in Cambridge, United Kingdom.

The programme included a summer school, a workshop, a series of presentations from key speakers, and a day devoted to interactions between academics and industry.
The event was attended by over 80 researchers from over 20 countries.
Details of the programme can be found on the web at:
http://www.newton.ac.uk/event/POP

The organizers of the event were Adam N. Letchford, Jean Bernard Lasserre, Markus Schweighofer and Jorg Fliege, with Monique Laurent and Kurt Anstreicher acting as international advisors.

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Semidefinite programming and eigenvalue bounds for the graph partition problem

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The graph partition problem is the problem of partitioning the vertex set of a graph into a fixed number of sets of given sizes such that the sum of weights of edges joining different sets is optimized.

In this paper we simplify a known matrix lifting semidefinite programming relaxation of the graph partition problem for several classes of graphs and also show how to aggregate additional triangle and independent set constraints for graphs with symmetry.

We present an eigenvalue bound for the graph partition problem of a strongly regular graph, extending a similar result for the equipartition problem.

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We also derive a linear programming bound of the graph partition problem for certain Johnson and Kneser graphs.

Using what we call the Laplacian algebra of a graph, we derive an eigenvalue bound for the graph partition problem that is the first known closed form bound that is applicable to any graph, thereby extending a well-known result in spectral graph theory.

Finally, we strengthen a known semidefinite programming relaxation of a specific quadratic assignment problem and the above-mentioned matrix-lifting semidefinite programming relaxation by adding two constraints that correspond to assigning two vertices of the graph to different parts of the partition. This strengthening performs well on highly symmetric graphs when other relaxations provide weak or trivial bounds.

Completely positive reformulations for polynomial optimization

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2 of 12 / Math. Program., Ser. B (2015) 151:405-431

There is a well established body of research on quadratic polynomial optimization problems based on reformulations of the original problem as a conic program over the cone of completely positive matrices, or its conic dual, the cone of copositive matrices. Novel solution schemes for quadratic polynomial optimization problems have been designed by drawing on conic programming tools, and the extensively studied cones of completely positive and of copositive matrices.

Along this line of research, we consider polynomial optimization problems. For this purpose, we use a natural extension of the cone of completely positive matrices; namely, the cone of completely positive tensors.

We provide a general characterization of the class of polynomial optimization problems that can be formulated as a conic program over the cone of completely positive tensors.

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As a consequence of this characterization, it follows that recent related results for quadratic problems can be further strengthened and generalized to higher order polynomial optimization problems.

They show that a wide family of PO problems can be reformulated as linear optimisation problems over completely positive cones. This extends a result of Burer, which applied only to certain mixed 0-1 quadratic programs.

Nesterov, Y.: Structure of Non-negative Polynomials and Optimization Problems. Technical Report 9749, CORE (1997)

Shor, N.: Class of global minimum bounds of polynomial functions. Cybernetics 23, 731–734 (1987)

 Math. Program., Ser. B (2015) 151:433-457

An alternative proof of a PTAS for fixed-degree polynomial optimization over the simplex

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The problem of minimizing a polynomial over the standard simplex is one of the basic NP-hard nonlinear optimization problems – it contains the maximum clique problem in graphs as a special case.

It is known that the problem allows a polynomial time approximation scheme (PTAS) for polynomials of fixed degree, which is based on polynomial evaluations at the points of a sequence of regular grids.

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In this paper, we provide an alternative proof of the PTAS property.

The proof relies on the properties of Bernstein approximation on the simplex. We also refine a known error bound for the scheme for polynomials of degree three.

The main contribution of the paper is to provide new insight into the PTAS by establishing precise links with Bernstein approximation and the multinomial distribution.

Certification of real inequalities: templates and sums of squares

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4 of 12 Math. Program., Ser. B (2015) 151:477-506

We consider the problem of certifying lower bounds for real-valued multivariate transcendental functions. The functions we are dealing with are nonlinear and involve semialgebraic operations as well as some transcendental functions like cos, arctan, exp, etc.

Our general framework is to use different approximation methods to relax the original problem into polynomial optimization problems, which we solve by sparse sums of squares relaxations.

We use a combination of max-plus algebra and sums-of-squares arguments to compute verifiable numerical proofs of validity for inequalities involving polynomials and/or transcendental functions.

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Narrowing the difficulty gap for the Celis-Dennis-Tapia problem

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5 of 12 Math. Program., Ser. B (2015) 151:459-476

We study the Celis-Dennis-Tapia (CDT) problem: minimize a non-convex quadratic function over the intersection of two ellipsoids.

In contrast to the well-studied trust region problem where the feasible set is just one ellipsoid, the CDT problem is not yet fully understood.

Our main objective in this paper is to narrow the difficulty gap that occurs when the Hessian of the Lagrangian is indefinite at all Karush–Kuhn–Tucker points.

We prove new sufficient and necessary conditions both for local and global optimality, based on copositivity, giving a complete characterization in the degenerate case. They give conditions for this problem to be polynomial-time solvable.

6 of 12 Math. Program., Ser. B (2015) 151:507-527

Tractable approximations of sets defined with quantifiers

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The author studies certain convex sets that are defined via polynomial inequalities and quantifiers. These have applications in robust optimisation. He shows how to construct tractable inner and outer approximations of these sets.

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Given a compact basic semi-algebraic set $K \subset R^n \times R^m$, a simple set B (box or ellipsoid), and some semi-algebraic function f , we consider sets defined with quantifiers, of the form

$$R_f := \{x \in B : f(x,y) \leq 0 \text{ for all } y \text{ such that } (x,y) \in K\}$$

$$D_f := \{x \in B : f(x,y) \geq 0 \text{ for some } y \text{ such that } (x,y) \in K\}.$$

The set R_f is particularly useful to qualify "robust" decisions x versus noise parameter y (e.g. in robust optimization on some set $\Omega \subset B$) whereas the set D_f is useful (e.g. in optimization) when one does not want to work with its lifted representation $\{(x,y) \in K : f(x,y) \geq 0\}$.
Assuming that $K_x := \{y : (x,y) \in K\} \neq \emptyset\}$ for every $x \in B$, we provide a systematic procedure to obtain a sequence of explicit inner (resp. outer) approximations that converge to R_f (resp. D_f) in a strong sense.

7 of 12 Math. Program., Ser. B (2015) 151:529-553

A semidefinite programming hierarchy for packing problems in discrete geometry

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Packing problems in discrete geometry can be modeled as finding independent sets in infinite graphs where one is interested in independent sets which are as large as possible.

For finite graphs one popular way to compute upper bounds for the maximal size of an independent set is to use Lasserre's semidefinite programming hierarchy.

We generalize this approach to infinite graphs. For this we introduce topological packing graphs as an abstraction for infinite graphs coming from packing problems in discrete geometry.

We show that our hierarchy converges to the independence number.

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Quartic spectrahedra

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Quartic spectrahedra in 3-space form a semialgebraic set of dimension 24.

This set is stratified by the location of the ten nodes of the corresponding real quartic surface.

There are twenty maximal strata, identified recently by Degtyarev and Itenberg, via the global Torelli Theorem for real K3 surfaces.

We here give a new proof that is self-contained and algorithmic. This involves extending Cayley's characterization of quartic symmetroids, by the property that the branch locus of the projection from a node consists of two cubic curves.

This paper represents a first step towards the classification of all spectrahedra of a given degree and dimension.

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The hierarchy of local minimums in polynomial optimization

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A combination of convex algebraic geometry and semidefinite programming to derive local optimality conditions for PO problems is used.

In some cases, these conditions enable one to enumerate local minima efficiently.

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This paper studies the hierarchy of local minimums of a polynomial in the vector space R^n.

For this purpose, we first compute H-minimums, for which the first and second order necessary optimality conditions are satisfied.

To compute each H-minimum, we construct a sequence of semidefinite relaxations, based on optimality conditions.We prove that each constructed sequence has finite convergence, under some generic conditions.

A procedure for computing all local minimums is given. When there are equality constraints, we have similar results for computing the hierarchy of critical values and the hierarchy of local minimums.

Approximate cone factorizations and lifts of polytopes

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Matrix factorisations and second order cone constraints are used to construct convex sets which lie in a space of polynomial dimension and which, when projected into an appropriate subspace, approximate any given polytope.

We show how to construct inner and outer convex approximations of a polytope from an approximate cone factorization of its slack matrix.

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This provides a robust generalization of the famous result of Yannakakis that polyhedral lifts of a polytope are controlled by (exact) nonnegative factorizations of its slack matrix.

Our approximations behave well under polarity and have efficient representations using second order cones.We establish a direct relationship between the quality of the factorization and the quality of the approximations, and our results extend to generalized slack matrices that arise from a polytope contained in a polyhedron.

Cutting planes for RLT relaxations of mixed 0-1 polynomial programs

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The reformulation-linearization technique, due to Sherali and Adams, can be used to construct hierarchies of linear programming relaxations of mixed 0-1 polynomial programs.

As one moves up the hierarchy, the relaxations grow stronger, but the number of variables increases exponentially.

We present a procedure that generates cutting planes at any given level of the hierarchy, by optimally weakening linear inequalities that are valid at any given higher level.

Computational experiments, conducted on instances of the quadratic knapsack problem, indicate that the cutting planes can close a significant proportion of the integrality gap.

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Exploiting symmetry in copositive programs via semidefinite hierarchies

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The authors consider PO problems whose difficulty arises due to their extreme symmetry. They show that approaches based on copositive representations can be speeded up significantly by handling the symmetry explicitly.

An important line of research is to use semidefinite programming to approximate conic programming over the copositive cone.

Two major drawbacks of this approach are the rapid growth in size of the resulting semidefinite programs, and the lack of information about the quality of the semidefinite programming approximations.

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These drawbacks are an inevitable consequence of the intractability of the generic problems that such approaches attempt to solve.

To address such drawbacks, we develop customized solution approaches for highly symmetric copositive programs, which arise naturally in several contexts.

For instance, symmetry properties of combinatorial problems are typically inherited when they are addressed via copositive programming.

As a result we are able to compute new bounds for crossing number instances in complete bipartite graphs.

Дякую за увагу!

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